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Accidental Encounters: Can Accidents be Adaptive?

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Abstract

Accidents happen in nature, from simple incidents like bumping into obstacles, to erroneously arriving at the wrong location, to mating with an unintended partner. Whether accidents are problematic for an animal depends on their context, frequency, and severity. In this paper, we investigate the question of how accidents affect the task performance of agents in an agent-based simulation model for a wide class of tasks called "multi-agent territory exploration tasks" (MATE). In MATE tasks, agents have to visit particular locations of varying quality in partially observable environments within a fixed time window. As such, agents have to balance the quality of the location with how much energy they are willing to expend reaching it. Arriving at the wrong location by accident, typically reduces task performance.

We model agents based on two location selection strategies that are hypothesized to be widely used in nature: *best-of-n* and *min-threshold*. Our results show that the two strategies lead to different accident rates and thus overall different levels of performance based on the degree of competition among agents, as well as the quality, density, visibility, and distribution of target locations in the environment. We also show that in some cases individual accidents can be advantageous for both the individual and the whole group.

Keywords

accidental encounters, partially observable environment, agent-based modeling

Introduction

Several natural tasks performed by animals require them to visit specific locations in their environment within a fixed amount of time. Foraging animals, for example, have to find locations that offer food before they run out of energy. Another example is searching for a mate where animals have to find mating partners during their breeding season. These and other related tasks can be seen as instances of "multi-agent territory exploration" (MATE) tasks (Schermerhorn & Scheutz, 2007b) which are defined by a group of agents A, a set of locations or "checkpoints" C, a search period T, and a (partially observable) environment E. Agents and checkpoints are distributed in the environment according to a distribution D, with each checkpoint having an associated "quality" which can be perceived by the agents. A typical goal in a MATE task would be to maximize the quality of visited checkpoints within T (although other goals can be specified as well).

Several variations of MATE tasks have been proposed in the literature. For example, agents might have to visit two locations consecutively such as finding food first and then bringing it back to a base location (Drogoul & Ferber, 1993), or they might have to meet up with mobile checkpoints (Araujo & Grupen, 1996). More complex variants still require multiple agents to visit checkpoints at the same time (Schermerhorn & Scheutz, 2007a).

In MATE tasks, accidental visitations of checkpoints can happen when an agent aims for a specific checkpoint, but accidentally stumbles across another checkpoint on its way, either because the agent ignored the checkpoint, or because the checkpoint was not visible to the agent. We will thus define an "accident" (in the MATE task) as "any visitation of an unintended checkpoint" – "unintended" here meaning "not selected based on the agent's choice strategy". An important question for MATE tasks thus is whether and to what extent accidents can have an impact on the average agent performance, since typical performance measures in MATE tasks require agents to visit the highest quality checkpoints possible. It seems *prima facie* intuitive that accidents could not be useful as they will likely only lower an agent's performance for any "reasonable" search strategy (i.e., one that attempts to maximize the checkpoint quality). However, as we will demonstrate in this paper, there are cases where accidents are beneficial for task performance.

Take, for example, the common instance of a MATE task where agents (e.g., females) attempt to find (stationary) mates located in the environment using different mate selection strategies (Baugh & Ryan, 2009) based on males' mating calls or display of prowess, which are indicative of the quality of the mate and thus a crucial determinant of the quality of the offspring (Welch, Semlitsch & Gerhardt, 1998). Since females typically only have partial knowledge of the location of possible partners, as some males may not advertise their location through calls, they may end up mating with non-optimal partners by accident (e.g., bumping into a potential mate causes females to simply mate

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with the male in some species such as treefrogs (Gerhardt, 1974)). For low quality males who might not win in a "shouting" competition with high quality males, it might thus be beneficial to remain silent and try to intercept females instead of calling, a strategy often referred to as "satellite" strategy (Leary, Fox, Shepard & Garcia, 2005); for if they decide to call, females might actively avoid them.

To explore the effects of accidental encounters on the performance of agents in MATE tasks, we use an agent-based model (ABM) which allows us to investigate the conditions under which accidents could be advantageous to the population. This model consists of agents choosing locations to visit according to two different selection strategies that have been hypothesized to be widely used in the animal kingdom: the *best-of-n* strategy (Janetos, 1980) and the *minimum-threshold* strategy (Jennions & Petrie, 1997). We will investigate how these strategies alter the frequency of accidents and what average quality change results from such accidents. For instance, we vary the ratio of agents, the distribution of agents and checkpoints in the environment, and the quality of the checkpoints, to determine how these independent variables interact with each other and affect the performance of agents.

The rest of the paper is structured as follows. We start by reviewing evidence for the importance of accidental encounters in different animals performing different tasks. Then we present various analytical models of behavior together with our own previous work investigating the performance of mating strategies in general, and accidental matings in particular. We then introduce the MATE task with the specific types of accidents shown by our model, followed by our experimental setup with independent and dependent variables. Next, we present the results of a large parameter sweep together with an analysis of the influence each parameter has on the frequency and average quality of accidental encounters. We also present a spatial analysis of the environment to show the distribution of accident locations. Then, we discuss our results, explaining why the two strategies overall reduce the likelihood of accidents (e.g., compared to a random strategy) and also showing when accidents can be advantageous for the population. We finish the paper by summarizing the main results and proposing extensions for future work.

Evidence for Accidental Encounters in Nature

Accidental encounters occur in nature in distinct contexts of social interactions. For example, it is known that chimpanzees can live in harmony with human populations; yet, some documented chimpanzee attacks on humans consisted of accidental encounters in areas frequented by both chimpanzees and humans (Hockings, Yamakoshi, Kabasawa & Matsuzawa, 2010). Although humans are not a common prey of chimpanzees, some of the attacks aimed at children showed predatory characteristics (McLennan, 2008; Wrangham, Wilson, Hare & Wolfe, 2000). These behaviors might lead to disease transmission from humans to chimpanzees, and are thought to negatively affect the population of primates, hence are an example of the negative effects of accidental encounters.

One particular area of interest in biology for studying the effects of accidental encounters is *accidental mating*. Females of gray treefrogs show phonotactic behavior, being attracted by the quality of male calls (Gerhardt, 1994). However, silent males have been found within groups of these frogs. The case of male "satellites" in amphibians, i.e., males that differ in size but not age from calling males and that remain silent instead of advertising their location through calls, is an example of a strategy that intrinsically explores accidents; for satellites can only mate when females "bump into them" by accident (Leary, Fox, Shepard & Garcia, 2005). Avoiding to call then seems to be a persistent tactic for smaller males that have a limited amount of energy to spend on calling and are not willing to engage in an aggressive interaction with larger males (Forester & Lykens, 1986). And, it turns out, that the strategy not only depends on the size of the individual male, but also on the structure and dynamics of the calling environment. For example, in breeding sites with many callers, lower quality callers spend more time acting as satellites than calling (Arak, 1983).

Another example for how population density can affect strategy choices that depend on accidents is the behavior of crickets during the mating season (Alexander, 1961). Females of some species of crickets are attracted by the sound emitted by males. However, in an environment configurations with a high density of crickets, there is a high probability of accidental intersexual encounters. Hence, fewer males decide to spend energy calling to attract females, an hypothesis confirmed by results from another study (Hissmann, 1990) where a population of crickets of species *Gryllus campestris* was observed for a long period of time. During the observation time, the population density decreased and the sex ratio showed a high amount of males in the environment. At high density, fewer males called to attract females and surprisingly the silent males proportionally had more mates than the calling ones. These encounters between females and silent males might have been accidental. As the population density started to reduce, the probability of mating by chance also decreased. Consequently, males had to compete for the few remaining females and, hence, more males had to call in order to attract those females.

Often, individuals of different species share the same mating site and their breeding season also overlaps. This is the case with two species of treefrogs *Hyla cinerea* and *Hyla gratiosa*. Although these two species are interfertile, females prefer conspecific calls (i.e., moving towards males of the same species) (Mecham, 1960). However, heterospecific pairs were observed in field (Oldham & Gerhardt, 1975). Oldham and Gerhardt suggested that accidental matings between individuals of different species more often happen in cases where males call from areas close to water. Therefore, calling positions (i.e., distribution of males in the breeding sites), have significant influence on interspecies accidental encounters.

Accidental encounters have also been identified as the main form of mating in some species. For instance, researchers do not see relevant influence of courtship behaviors in mole salamanders since intersexual encounters happen, in general, by accident (Verrell & Krenz, 1998). Verrell and Krenz proposed two experiments in order to compare how males perform when they need to compete for a female with only one male and with three other males, respectively. The goal of this study was to verify the males' mating efforts according to different operational sex ratios. The authors hypothesized that if accidents were not relevant for mating success, males would court more when there were more competitors. They found that males of mole salamanders decrease their mating effort when there are more males competing for the same female (i.e., there is a smaller probability of mating). Verrell and Krenz believe that this strategy might have been evolved because males arrive at breeding sites earlier in the mating season, when there is more competition for mates. Therefore, males prefer to wait until later in the season when there is a higher chance of accidentally finding competition-free encounters with females. Consequently, a higher probability of accidental encounters is beneficial for the males in the population.

While accidents clearly happen in nature and different explanations have been proposed on a case-by-case basis for why they might be useful or adaptive, there is currently no computational study that systematically investigates the tradeoffs between different mating strategies and accidents. Yet, such a study would be important for providing the necessary theoretical insights that can put the above findings on a more formal footing and provide more concise explanations for the advantages and disadvantages of accidents.

Background and Previous work

Mathematical and computational models have been frequently proposed in the literature to explain animal behaviors such as mating strategies. In general, these models aim for evolutionary explanations, trying to discover strategies that evolved in populations through a long timespan (e.g., *evolutionary stable strategies*). In the context of mating, there are models about conflicts over resources (Houston & McNamara, 1988); understanding when to call and when to forage (McNamara, Mace & Houston, 1987; Thomas & Cuthill, 2002); mating selection strategies (Janetos, 1980; Jennions & Petrie, 1997; Real, 1990); modeling satellite behavior (Lucas, Howard & Palmer, 1996; Arak, 1988; Waltz, 1982). However, all these models contain a high number of parameters, and finding the complex interactions among all those parameters is typically not feasible. Moreover, for some tasks, the spatial distribution of elements can be relevant for the success of agents (Collins, McNamara & Ramsey, 2006), which often is not captured in analytic solutions.

Agent-based models (ABMs) use the individual interactions among agents and the environment to show how global population-level behaviors can arise from these individual interactions. Thus, they are often more useful for behavioral studies as they (1) enable the possibility of testing and explaining different characteristics and dynamics of agents and their environments, and (2) allow researchers to observe how changes in parameterization affect interactions of agents and thus task performance.

Often, researchers analyze agents' performance in MATE tasks using an absolute performance. However, "performance-cost-tradeoffs" are often more informative. In (Schermerhorn & Scheutz, 2007b), for example, we investigated the cost-benefit of four candidate architectures for a MATE task. We varied agents' attributes such as sensory range, group size and behavior prediction abilities of agents and showed that the relative performance of the architectures differed with the costs assigned to each agents' attributes.

Some MATE tasks require coordination. For example, in a mating task in which both partners must be present at a specific location (e.g., a site to lay eggs), males and females must coordinate in order to satisfactorily perform the task. In general, when agents are allowed to communicate, their performance in the task is improved. But in (Schermerhorn & Scheutz, 2007a), we showed that the cost of communication is relevant to decide whether the agents benefit from communicating.

In our previous work, we were interested in exploring how different strategies in the MATE task perform in different biologically plausible configurations, focusing on mating strategies of the grey treefrog *Hyla versicolor* (Scheutz, Harris & Boyd, 2010). Results from simulations of a short period (one hour) in the task can be easily compared with empirical data and can therefore generate meaningful biological explanations. We compared two strategies for mate selection: *best-of-closest-n* – "best-of-n" for short – and *closest-above-minimum-threshold* – "min-threshold" for short – investigating whether one strategy dominated another. *Strategy domination* means that one strategy consistently leads to significantly better performance than the other strategy for a given parameter range. We found that even though *min-threshold* performance was better than *best-of-n* on the majority of the parameter space, there were regions in our parameter space where the performance of *best-of-n* surpassed *min-threshold*. Therefore, *min-threshold* did not dominate *best-of-n*.

We also started a preliminary exploration of accidental matings in fully observable environments and showed that accidents decrease the average quality of the mated males (Ferreira & Scheutz, 2015). Comparing the rate of accidents and the quality of accidentally mated males with the rate and quality of non-accidental matings, we found that the *best-of-n* strategy had an undesirable higher rate of accidents compared to the *min-threshold* strategy. However, accidents in *min-threshold* had a more detrimental influence on the quality of mated males because females playing *min-threshold* only moved toward a male when he was "worth it" (i.e., males with quality above threshold). While rarer, accidents with males below the threshold drastically reduced the average quality of mated males. Overall, it is an open question whether accidents that led to worse performance in the short-term (i.e., the mating task), might have important long-term effects (e.g., to maintain the diversity in the gene pool).

In this paper, we will extend the above study to a more general class of tasks and partially observable environments, and again analyze the frequency of accidents as well as the influence of those accidents on the average quality of the visited checkpoints. Table 1 shows how MATE's entities (agents and stationary as well as mobile checkpoints) can be mapped to a few different biological tasks.

In addition to investigating MATE tasks with larger parameter spaces, we specifically explore how different

MATE	Agents	Stationary Checkpoints	Mobile Checkpoints
Foraging Task	Animals	Food	Prey
Mating Task in Treefrogs	Females	Calling Males	
Mating Task in Salamanders	Females		Males

Table 1. Relation between MATE and biological agents and tasks.

checkpoint distributions change the rate and quality of accidents (previously we had only used a Gaussian distribution of checkpoints). Moreover, to address the previous simplification that all male frogs started in the environment from the beginning, in this paper we allow new checkpoints to appear in the environment as time goes on.

Model for Multi-agent Territory Exploration Task (MATE)

We modeled a general MATE task with stationary checkpoints to compare the *efficiency* of different checkpoint selection strategies (i.e., how fast agents found and visited the best quality checkpoints (Schermerhorn & Scheutz, 2007a)). Agents start from the edges of the environment and have to decide which location to move towards.

Agents update their states in two distinct phases: sensing and acting. During the sensing phase, agents sense the qualities of checkpoints and select candidate checkpoints according to either *best-of-n* or *min-threshold* strategy. In *best-of-n*, agents select the best checkpoints from a set of n closest checkpoints. In *min-threshold*, agents select the closest checkpoint with quality above a minimum threshold θ. After selecting a checkpoint, agents move in a straight line towards the chosen checkpoint. When an agent reaches the desirable checkpoint, that checkpoint is removed from the environment. At the next cycle, the agent will choose another checkpoint during sensing phase.

We start by formally defining the two main strategies (see also (Ferreira & Scheutz, 2015; Scheutz, Smiley & Boyd, 2013)). Let C_i be the set of all checkpoints in the environment at a cycle i, and let A be the set of all agents in the environment. Also let l_q be the quality of the checkpoint $l \in C_i$. Finally, let $D(a, f)$ denotes the straight line distance between an agent a and a location l. Define the set of closest checkpoints from a given set X to a given agent a as $c(a, X) = \{j \in X | \nexists k \in X[D(a, k) < D(a, j)]\}$ and let $c^n(a, X)$ denotes the set of n closest from set X with respect to the location of agent a.

- *best-of-n*. The selected checkpoint at cycle i is $argmax_{l \in c^n(a,C_i)}(l_q)$ for the agent a, i.e., the goal checkpoint with highest quality in the set of closest n checkpoints.
- *min-threshold*. The selected checkpoint at cycle i is $argmax_{l \in c(a,\{g \in C_i | l_q \geq f_\theta\})}(l_q)$, where f_θ is the minimum threshold of agent a, i.e., the checkpoint with the highest quality above the minimum threshold among the closest checkpoints.

Selection strategies balance the quality of checkpoints and cost to reach them. However, often agents visit undesirable checkpoints. These accidental encounters can increase the number of visited checkpoints and at the same time, as we showed in (Ferreira & Scheutz, 2015), reduce the average quality of visited checkpoints. Thus, the goal of this general MATE model is to explore the conditions in which accidental encounters are more likely to happen and the effects of these accidents on the performance of the agents. A detailed description of the model according to the ODD (Overview, Design concepts, Detail) protocol of agent-based models (Grimm, Berger, DeAngelis, Polhill, Giske & Railsback, 2010) can be checked in the appendix.

Types of Accidents

Due to the different configurations we tested, three different types of accidents might occur. Type 1 accidents happen between an agent and a checkpoint that displays its quality (i.e., non-satellite) in the path towards another checkpoint of better quality. Figure 1a shows an example of Type 1 accidents. This is the most common type of accidental encounter and can exist in any configuration we tested. Moreover, this type of accident always reduce the average quality of visited checkpoints of one specific agent because if the quality is higher than the original choice, than the accident would be chosen. A detailed proof of this statement can be seen in (Ferreira & Scheutz, 2015).

Type 2 accidents happen only when $c_p = progressive$. In these accidents, the agent is moving towards a specific checkpoint but before reaching the selected checkpoint, another checkpoint spawns within $d_{visiting}$ to an agent, effectively forcing the agent to visit it right away. the agent. Type 2 accidents can increase or reduce the average quality of visited checkpoints because the new checkpoint can have any quality.

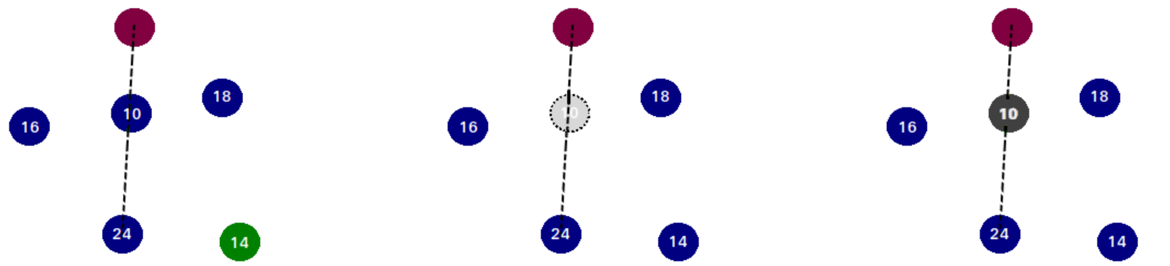
Finally, Type 3 accidents happen between agents and satellites. Satellites are the checkpoints of worst quality and only exist in $\omega = partially - observable$. Therefore, Type 3 accidents also reduce the average quality of accidents. Satellites do not display their qualities, hence agents can not sense and avoid them. Figure 1c shows an example of Type 3 accidents.

Experimental Setup and Hypotheses

We modeled the MATE task in our agent-based simulation environment *Simworld* (Scheutz & Harris, 2011) and added mechanisms to record the number and distributions of accidental encounters.

We fixed environmental parameters such as the size of the environment as 20m x 25m, the termination condition $Term$ at 3600 cycles (one cycle corresponding to one second of real-time) and $d_{visiting} = 4cm$ (within which checkpoints are considered visited). We also fixed m_δ as a uniform distribution (i.e., we placed agents near the edges and every point had the same probability of containing an agent). Moreover, we fixed the standard deviation of checkpoint quality $\sigma_q = 2$. Finally, we fixed the velocity of agent movements at $a_v = 1.86cm/s$.

We varied the remaining parameters in a systematic way to investigate their possible interactions with the number and



(a) Type 1 Accident **(b)** Type 2 Accident **(c)** Type 3 Accident

Figure 1. Examples of accidental encounters showed by our simulation. Red circle represents an agent. Circles with numbers represent checkpoints with their qualities. The agent is using *best-of-n* with $n = 4$. The dashed line is the path the agent will traverse. (a) the agent senses the 4 closest checkpoints (blue circles) but can not sense the checkpoint 16 (checkpoint 16 is the farthest), and selects checkpoint 24. On the agent's path towards checkpoint 24, it accidentally visits checkpoint 10. (b) simulation with $c_p = progressive$ and checkpoint 10 has not emerged yet. If checkpoint 10 is placed when the agent is close to checkpoint's location, an accidental encounter happens. (c) simulation with $\omega = partially - observable$ and checkpoint 10 is a satellite that does not display its quality. The agent senses the other four checkpoints and selects checkpoint 24. On the path towards checkpoint 24, the agent accidentally visits the satellite.

quality of accidents. We varied the number of agents $m_n \in \{1, 5, 10, 15, 20\}$. We expect a positive correlation between m_n and the ratio of accidents, because increasing the number of agents wandering around increases the probability of those agents accidentally visiting an undesirable checkpoint. On the other hand, we do not expect any correlation between the number of agents and the average quality of accidents.

We also varied the checkpoint placement method c_p in *simultaneous* and *progressive*. We expect more accidents in simulations with *simultaneous* compared to *progressive* placement, because there exists a higher checkpoint density when agents exist since the beginning of the simulation. However, we do not expect any difference in the quality of accidents between both placement methods. We have the same hypothesis about the number of checkpoints in the environment c_n as increasing this number consequently will increase the density of checkpoints. Thus, we varied $c_n \in \{100, 200, 300, 400, 500\}$ to test our hypothesis about increased accident rates using three distribution of checkpoints c_δ: *uniform*, *Gaussian* and *inverseGaussian*. In the *uniform* distribution, all areas in the environment have the same probability of becoming a checkpoint. In the *Gaussian* distribution, more checkpoints exist in the central area of the environment. In the *inverseGaussian* distribution, more checkpoints exist near the edges of the environment. We expect a significant interaction between these distributions and the rate of accidents. More specifically, we expect that *uniform* distribution would lead to fewer accidents because checkpoints are not "clustered" in an area. Therefore, agents could move to the selected checkpoint with reduced probability of accidentally visiting another checkpoint. We do not expect any relevant difference between the quality of accidents for each distribution.

We considered three selection strategies $best - of - n$, $min - threshold$ and a $random$ choice. For $best - of - n$ we varied its parameter $n \in \{1,\ 2,\ 3,\ 4,\ 5\}$; for $min - threshold$ we varied $\theta \in \{6,\ 12,\ 18,\ 24\}$; the *random* choice strategy is not parameterized. In (Ferreira & Scheutz, 2015) we showed that strategies reduced accidents. Moreover, we showed that $min - threshold$ led to fewer accidents than $best - of - n$. However, accidents in $min - threshold$ drastically decreased the average performance of the population. The same results are expected here.

Finally, we also varied the mean quality of the checkpoints $\mu_q \in \{6,\ 12,\ 18,\ 24\}$. We previously showed that the quality of the checkpoints interacts with the strategies in a complex fashion. While agents using $best - of - n$ do not take the absolute quality of checkpoint into consideration when they make a choice, agents using $min - threshold$ only approach checkpoints with quality above θ. Thus, we expect that configurations with high values of μ_q would display fewer accidents because agents would move for shorter distance, hence reducing the probability of accidentally bumping into an undesirable checkpoint.

To examine whether satellite behavior might be adaptive, we tested all previous combinations of parameters with both values of ω. With $\omega = fully - observable$, all checkpoints display their qualities. Forester and Lykens reported a maximum of 14% of satellites in a population of frogs (Forester & Lykens, 1986). Thus, in the second condition ($\omega = partially - observable$), we fixed the number of satellites at 10%, i.e., the 10% worst quality checkpoints do not display their qualities and are thus not visible to agents selecting checkpoints.

Due to the stochastic characteristics of the model, we ran 100 simulations with distinct initial conditions for each point in the parameter space for a total of 1,200,000 simulations. The dependent variables were the number of visited checkpoints, the number of accidentally visited checkpoints, and the average quality for both accidentally and non-accidentally visited checkpoints.

We limited the range of the tested parameters due to computational constraints. The first hypothesis we wanted to explore was the influence of agent-checkpoint ratio on the number of accidents. Thus, as we did not know how the data were distributed, we uniformly varied c_n and m_n (it does not make sense to have 0 agents, thus we used a minimum of 1 agent). Even though we are proposing a general MATE simulation, we decided the upper bound of the parameter n based on empirical data in mating experiments with frogs (Jennions & Petrie, 1997). We decided to match the values of μ_q and θ to compare different interactions between both parameters.

We aim at drawing general conclusions of the influence of accidents in MATE tasks. For example, when we introduce a partially-observable environment, satellites could be "bad callers" in the mating task performed by treefrogs, or satellites could be hidden predators in a foraging task. The checkpoint placement methods represent, for example, whether all possible mates are present in the environment since the beginning of the mating season, or they enter the environment in different times. We also test distinct checkpoint distributions because animals congregate differently during the mating season. While some animals create clusters of possible choices, other are more uniformly distributed across the mating environment. The same rationale works for food in a foraging task.

Different tasks have different agent-checkpoint ratios. Foraging tasks have much more checkpoints than mating tasks. But, as we presented in the "Evidence of Accidental Encounters in Nature", even different mate tasks have different agent-checkpoint ratios. Thus, we decided to vary the number of agents and the number of checkpoints to get a wide range of possible agent-checkpoint values.

Moving checkpoints could have been added to our MATE task. However, with moving checkpoints there are several parameters that would have to be determined regarding the movement (e.g., direction/trajectory, speed, what to do when colliding, etc) which would either significantly increase the number of simulation runs (and make some parameter explorations not feasible) or require assumptions (about the various motion parameters) that might be difficult to justify for the general case. Hence, it might be better to investigate moving checkpoints for particular MATE tasks (e.g., particular predator-prey simulations).

Results

The results of the parameter sweep showed significant main effects of all variables on the frequency and quality of accidents. The two non-random selection strategies overall reduced the number of accidents in addition to maintaining a better performance than the random selection. The number of agents and checkpoints affected the number of accidents and there were differences on the dynamics of accidents according to when new checkpoints appeared in the environment and what their mean quality was. Furthermore, some areas of the environment were more likely to have accidents, hence would be good locations for satellites to increase the chance of being accidentally visited.

Which is the Best Strategy for Agents?

Depending on the particular instance of the MATE task, one of two goals would be preferable: maximization of the number of visited checkpoints (e.g., for foraging for food), or maximization of the quality of visited checkpoints (e.g., for finding a suitable mate). The results show that overall accidents increase the number of visited checkpoints, but often reduce the average quality of collected checkpoints.

Table 2 shows means and standard deviations of the number and quality of visited checkpoints, including or not the ones visited by accident. Agents following the *best-of-n* strategy visit more checkpoints and have more accidents on average. On the other hand, *min-threshold* is the strategy by which agents are more selective. Agents following this strategy only spend energy in moving when there are good quality checkpoints in the environment. Moreover, comparing both strategies with the *random* baseline strategy, we can see that although agents following *best-of-n* visit more checkpoints than in the random case, the number of accidents is much lower in *best-of-n* than random (Figure 2). This confirms our previous findings that agents using strategies have fewer accidents (Ferreira & Scheutz, 2015). Comparing *best-of-n* with *min-threshold*, the number of accidents is smaller in *min-threshold* because agents traverse shorter distances and therefore had a smaller probability of accidentally visiting an unintended checkpoint.

Looking at the quality of visited checkpoints, strategies increase the average quality of visited checkpoints. Moreover, agents using the *random* strategy display the best average quality of accidents because those accidents have random quality, which means that the average quality of accidental and non-accidental visits are not significantly different. When we compare the other two strategies, we can see that being highly selective pays off: The average quality of visited checkpoints in *min-threshold* is much higher than in *best-of-n*. However, the average quality of accidents is much lower in *min-threshold* than in *best-of-n*, thus when an accident happens with agents following *min-threshold*, the effect on the overall task performance is more severe than in *best-of-n* (Figure 3).

Is There a Significant Effect of Accidents on Selection Strategies?

Accidents increase the number of visited checkpoints and often reduce the average quality of visited checkpoints. Yet, it is necessary to compare the influence of accidents on the performance of different strategies. We performed paired-samples t-test to compare the number of visited checkpoints with accidental encounters versus without accidental encounters for all strategy and parameters (Table 3). There were significant differences in the number of visited checkpoints with and without accidents for all strategy and parameters. These results suggest that a significant part of the checkpoints was visited by accident. Specifically, accidental encounters increase the number of visited checkpoints on all selection strategies and for all parameters.

Figure 4 depicts the average number of accidents for simulations with and without accidents for all strategies. The highest number of accidents happened with the *random* choice. When we compare the other two strategies, agents using *best-of-n* visited more checkpoints by accident on average than agents using *min-threshold*.

Table 4 shows the results of paired-samples t-test to compare the quality of visited checkpoints with accidental encounters versus without accidental encounters conditions for all strategies and parameters. Again, there were significant differences for all strategy and parameters. These results indicate that accidental encounters are detrimental to the performance of all three selection

Strategy	Number: Visited Check-points	Number: Visited Check-points Without Accidents	Number: Visited by Accident	Quality: Visited Check-points	Quality: Visited Check-points Without Accidents	Quality: Visited by Accident
random	68.93 ± 48.50	49.29 ± 32.63	19.64 ± 20.11	15.14 ± 6.72	15.20 ± 6.72	14.94 ± 6.76
best-of-n	153.42 ± 108.16	148.76 ± 105.16	4.66 ± 5.09	15.74 ± 6.73	15.79 ± 6.73	14.13 ± 6.79
best-of-1	182.42 ± 119.84	182.14 ± 119.68	0.28 ± 0.66	15.18 ± 6.71	15.19 ± 6.71	11.64 ± 6.75
best-of-2	162.80 ± 111.80	158.02 ± 108.07	4.78 ± 4.34	15.57 ± 6.71	15.61 ± 6.71	14.04 ± 6.77
best-of-3	149.81 ± 105.03	144.04 ± 100.56	5.76 ± 5.10	15.81 ± 6.72	15.87 ± 6.72	14.26 ± 6.76
best-of-4	140.06 ± 99.60	133.95 ± 94.90	6.11 ± 5.32	15.99 ± 6.72	16.06 ± 6.72	14.34 ± 6.76
best-of-5	132.01 ± 95.43	125.64 ± 90.52	6.37 ± 5.54	16.14 ± 6.72	16.22 ± 6.72	14.42 ± 6.77
minthresh	99.94 ± 115.24	98.91 ± 114.92	1.02 ± 2.08	18.15 ± 5.86	18.58 ± 5.69	13.58 ± 6.21
minthresh 6	167.94 ± 113.76	167.11 ± 113.55	0.83 ± 1.67	15.45 ± 6.36	15.47 ± 6.34	8.65 ± 6.22
minthresh 12	122.87 ± 118.82	121.71 ± 118.76	1.16 ± 2.20	17.14 ± 5.35	17.61 ± 4.69	10.92 ± 4.61
minthresh 18	77.27 ± 106.47	76.18 ± 106.03	1.09 ± 2.20	20.36 ± 3.55	21.01 ± 2.62	15.51 ± 2.94
minthresh 24	31.68 ± 66.35	30.66 ± 65.11	1.01 ± 2.21	23.87 ± 2.26	24.93 ± 0.60	20.64 ± 2.18

Table 2. Summary of means and standard deviations for number and quality of visited checkpoints, visited checkpoints without the ones visited by accident and visited by accident for each strategy and parameter.

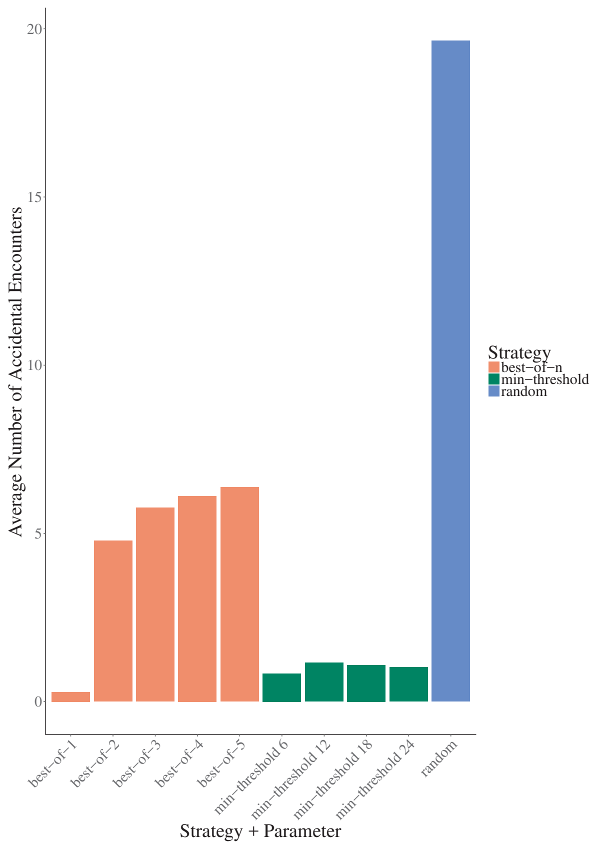


Figure 2. Effects of different strategy parameters on the number of accidental encounters. The bars represent the average number of accidental encounters for all simulations runs for each strategy plus parameter.

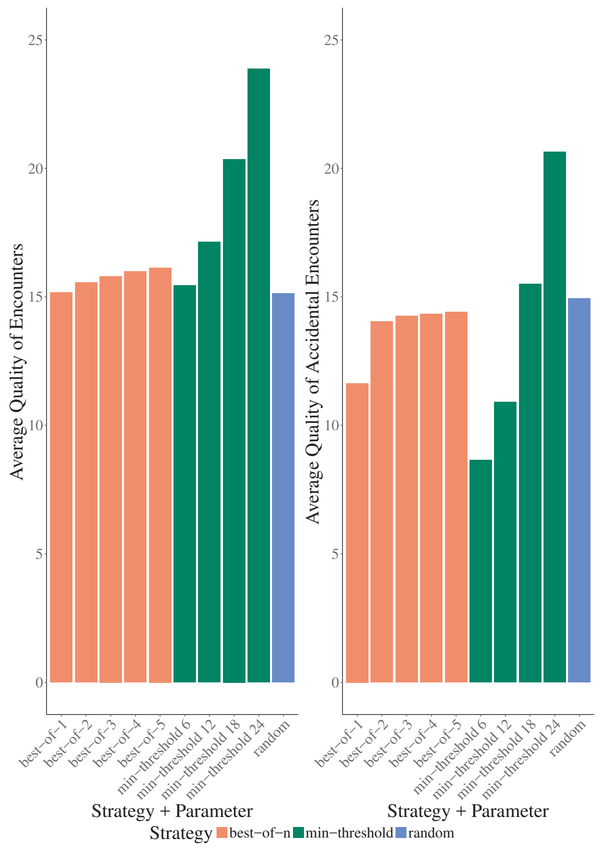


Figure 3. Effects of different strategy parameters on the quality of visited checkpoints and accidentally visited checkpoints. The bars represent the average quality of checkpoints visited by agents for all simulations runs and for each strategy plus parameter.

strategies (i.e., accidental encounters reduce the quality of visited checkpoints on all selection strategies and for all parameters).

Figure 5 shows the average quality of visited checkpoints with and without accidents. Accidents reduced the quality of visited checkpoints in all selection strategies. Moreover,

Strategy	t	df	p-value
random	338.42	12000	<2.2e-16
best-of-n	708.98	600000	<2.2e-16
best-of-1	148.6	120000	<2.2e-16
best-of-2	381.55	120000	<2.2e-16
best-of-3	391.77	120000	<2.2e-16
best-of-4	397.67	120000	<2.2e-16
best-of-5	398.26	120000	<2.2e-16
minthresh	340.07	480000	<2.2e-16
minthresh 6	172.88	120000	<2.2e-16
minthresh 12	182.52	120000	<2.2e-16
minthresh 18	170.98	120000	<2.2e-16
minthresh 24	159.47	120000	<2.2e-16

Table 3. Paired-samples t-test to compare the number of visited checkpoints in conditions where all encounters were accounted and the same simulations where only non-accidental encounters were accounted.

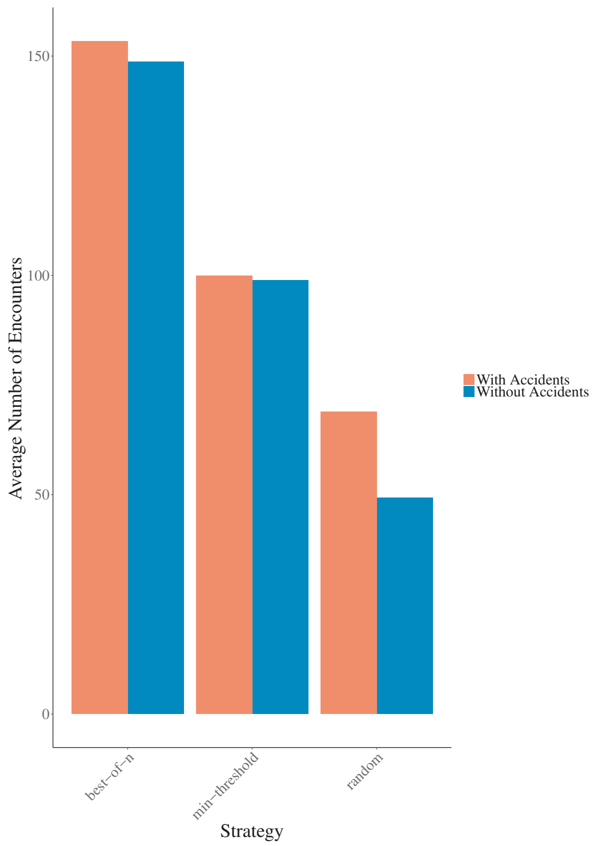


Figure 4. Effects of accidents on the number of encounters for each selection strategy. The bars represent the average number of checkpoints visited by agents for all simulations runs and for each strategy.

accidents hurt the performance of agents using the *min-threshold* the most. Thus, even though the number of accidents in *min-threshold* is the lowest, these accidents have a big influence on the average quality of visited checkpoints.

Strategy	t	df	p-value
random	-81.939	12000	<2.2e-16
best-of-n	-747.56	600000	<2.2e-16
best-of-1	-128.96	120000	<2.2e-16
best-of-2	-398.3	120000	<2.2e-16
best-of-3	-431.57	120000	<2.2e-16
best-of-4	-442.57	120000	<2.2e-16
best-of-5	-449.22	120000	<2.2e-16
minthresh	-198.05	347460	<2.2e-16
minthresh 6	-175.3	120000	<2.2e-16
minthresh 12	-113.39	105820	<2.2e-16
minthresh 18	-115.67	75821	<2.2e-16
minthresh 24	-122.39	45821	<2.2e-16

Table 4. Paired-samples t-test to compare the quality of visited checkpoints in conditions where all encounters were accounted and the same simulations where only non-accidental encounters were accounted.

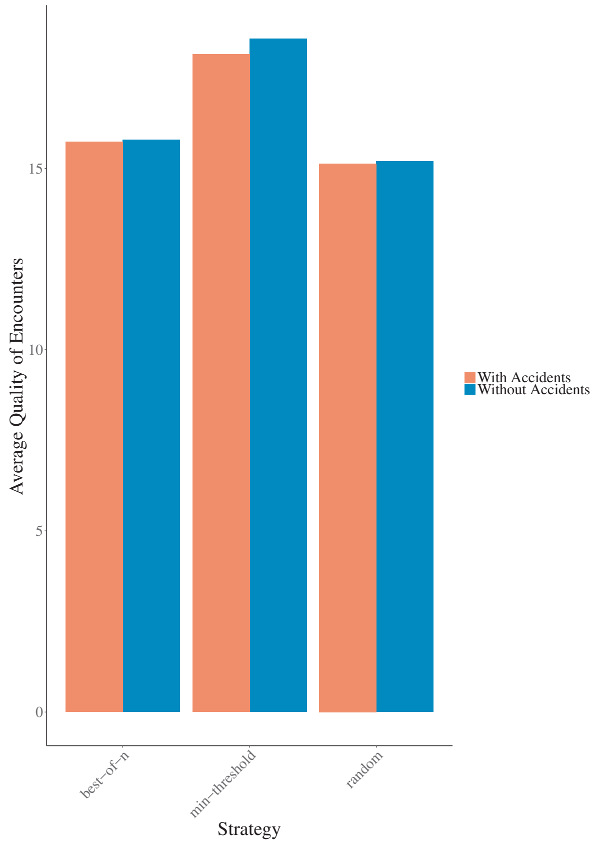


Figure 5. Effects of accidents on the quality of encounters for each selection strategy. The bars represent the average quality of checkpoints visited by agents for all simulations runs and for each strategy.

What is the Influence of the Agent-Checkpoint Ratio on the Number of Accidents?

We changed agent competition in two forms. We varied the number of agents – more agents led to more competition – and we varied the number of checkpoints in the environment – fewer checkpoints led to more competition. Increasing the number of agents increased the number of accidents in all strategies. Figure 6 shows that the number of accidents in agents using the *random* strategy has the highest increase. The other two strategies show a smaller slope. The reason why the number of accidents with the two non-random strategies does not increase at the same ratio as the number of agents is the following: Consider an agent a moving towards a checkpoint c_1 that would lead to an accident with another checkpoint c_2. If there are more agents in the environment, the probability of a different agent reaching c_2 before an accident can occur is higher, thus reducing the likelihood of this accident happening.

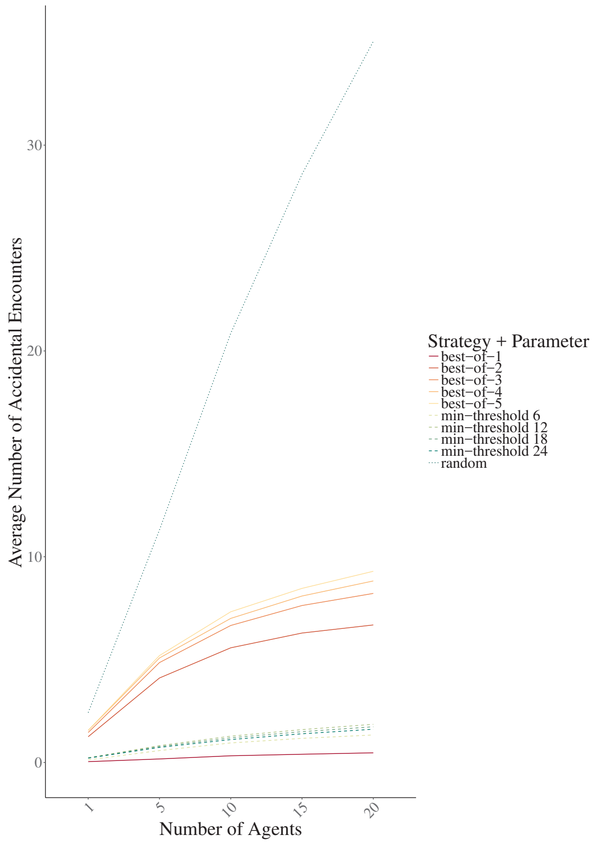


Figure 6. Effects of different selection strategies on the number of accidental encounters for each number of agents. The line represents the average number of accidental encounters at each simulation.

Figure 7 shows the interaction between the number of checkpoints and the number of accidents for each strategy. Accidents are more frequent as the number of checkpoints increase with all strategies. The reason is that more checkpoints in the environment lead to more "clusters" where accidents are more likely. Comparing all strategies, agents using the *random* or *best-of-n* show a greater increase of accidents than agents using *min-threshold*. This is because in configurations with more checkpoints, agents following the *min-threshold* strategy have to move shorter distances to find a checkpoint with quality higher than θ, which reduces the likelihood of accidents.

How Does the Moment of Appearance Change the Accidents?

In our experiments, all checkpoints either exist at the beginning of the simulation ($c_p = simultaneous$) or they

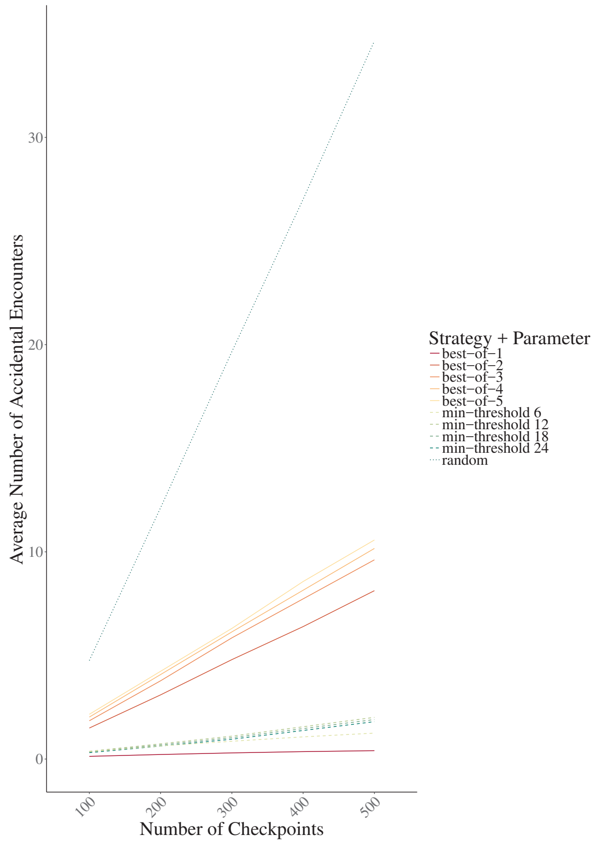


Figure 7. Effects of different selection strategies on the number of accidental encounters for each number of checkpoints. The bars represent the average number of accidental encounters at each simulation.

emerge uniformly as time passes ($c_p = progressive$) through progressive placement which gradually increases the density of checkpoints in the environment. Consequently, the probability that more than one checkpoint would exist on a straight line is lower. Figure 8 shows that the simultaneous placement leads to more accidents than the progressive placement method. In (Ferreira & Scheutz, 2015), we showed that agents using *best-of-n* strategy with $n = 1$ can not have any accidents. This is not true in the current setting for two reasons: first, in the progressive placement, there is a very small probability that a Type 2 accident exists. Second, in configurations with $\omega = partially - observable$, Type 3 accidents might exist with *best-of-1* strategy.

Figure 9 depicts the effects of strategy parameters and the placement methods on the quality of accidents. At first glance, it is not obvious why the placement methods change the quality of accidental encounters. As all checkpoints appear at the same locations and with the same quality, one would assume that the quality of accidents would not change. However, what really happens is that in the progressive placement, Type 2 accidents might exist, even though this accident is not always detrimental.

More specifically, if a checkpoint with quality d emerges near an agent pursuing another checkpoint with quality q and $d > q$, then the agent would accidentally visit this new checkpoint and the average performance would increase.

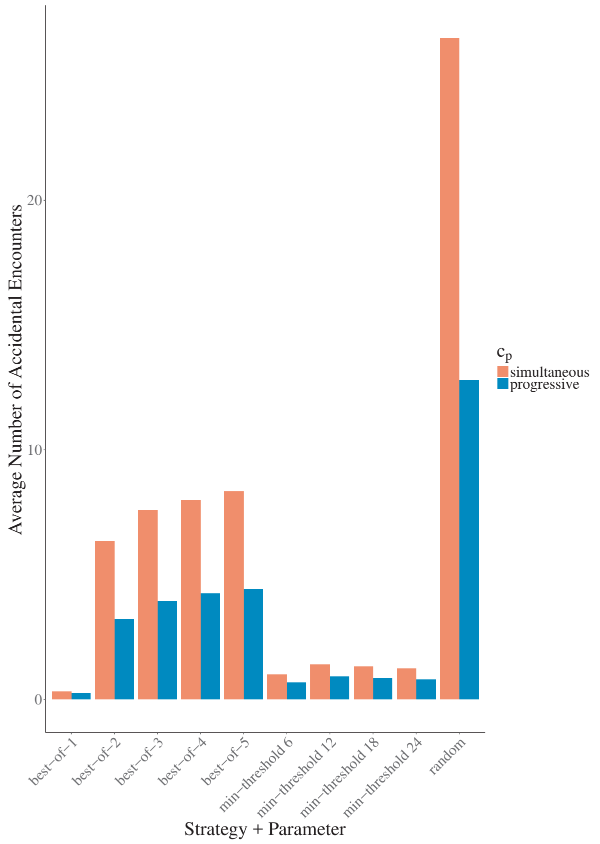


Figure 8. Interaction between strategy parameters and the different placement methods on the number of accidents. The bars represent the average number of accidental encounters for all simulation runs for distinct strategy plus parameter pairs for each placement method.

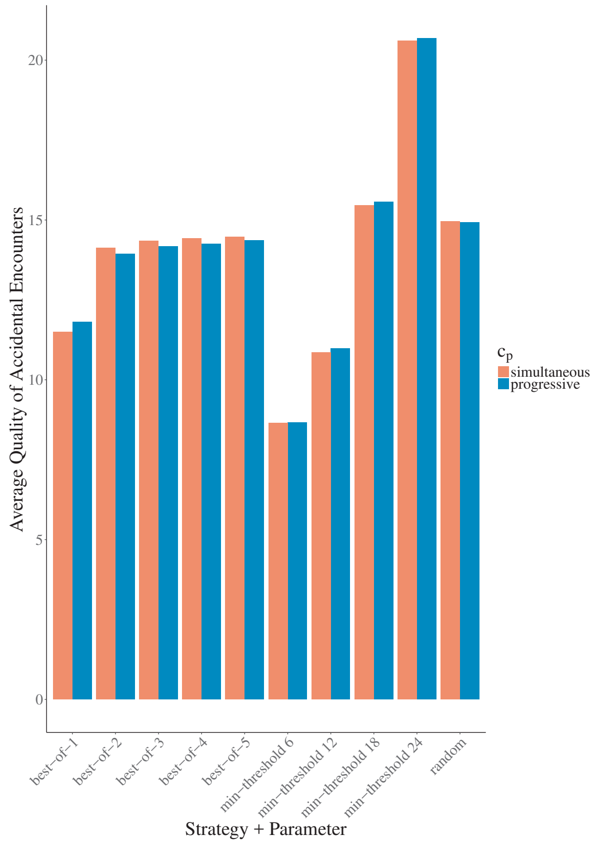


Figure 9. Interaction between strategy parameters and the different placement methods on the average quality of accidentally visited checkpoints. The bars represent the average quality of accidental encounters for all simulation runs for distinct strategy plus parameter pairs for each placement method.

Does the Quality of Checkpoints Change the Dynamics of Accidents?

The quality of checkpoints is relevant for the number of accidents only in configurations with agents using *min-threshold* strategy. Figure 10 shows the average number of accidents for all strategies and mean quality of checkpoints. Agents using *best-of-n* are not influenced by the average quality of checkpoints. They sample the closest checkpoints and move toward the best among them, independent of its quality.

On the other hand, agents playing the *min-threshold* strategy only move toward checkpoints above a minimum threshold. Hence, if the average quality is much lower than the minimum threshold of agents, then those agents decide not to move. As a result, agents do not visit any checkpoint, neither on purpose nor by accident. At the other side of the spectrum, if the average quality of checkpoints is much higher than the minimum threshold, then the agents always move toward the closest agent (same behavior as best-of-1). Accidents in these configurations happen with invisible checkpoints (i.e., accidents of Type 2 or Type 3).

Finally, when the average quality of checkpoints is close to θ, there are some checkpoints above threshold that are worth pursuing and there are also checkpoints below threshold that can exist in a location closer to the agents than those above threshold (the latter could become an accidental encounter). In our experiments we verified that the highest number of accidents in *min-threshold* happened in simulations which $\theta = \mu_q$. In these cases, half of checkpoints are below threshold and might lead to an accident. Hence, there is a high chance of an agent visiting one of these checkpoints. In simulations where $\theta = \mu_q + 3 \times \sigma_q$ (e.g., $\theta = 12 \wedge \mu_q = 6$), on average, only 2.1% of checkpoints are above threshold. Visitations to any other checkpoint count as an accident, hence the chance of accidents is the highest. However, when the agents finish visiting all good checkpoints they leave the environment. As they spend less time in the environment, the number of accidents is not as high as in configurations where $\theta = \mu_q$.

Moreover, the quality of accidents changes as the mean quality of checkpoints changes. Figure 11 shows that with the *best-of-n* strategy, as n increases, the quality of accidents remains approximately the same. On the other hand, with the *min-threshold* strategy, the parameter θ is an upper bound for the average quality of accidents. If the quality of checkpoints is much smaller than θ, agents following *min-threshold* do not spend energy on visiting checkpoints. Therefore, there is no accident. If θ is slightly higher than μq, then accidents hurt the performance the most because the quality of an accidental checkpoint is much lower than the other checkpoints the agents have been visiting (i.e., checkpoints with quality above θ). If $\theta = \mu q$ then the accidents do not

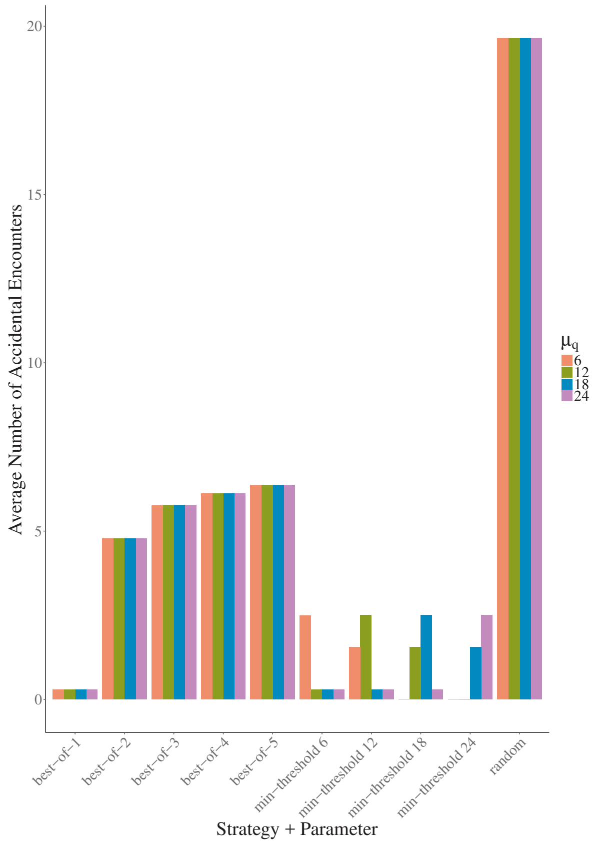


Figure 10. Effects of different selection strategies on the number of accidental encounters for each mean quality of checkpoints. The bars represent the average number of accidental encounters at each simulation.

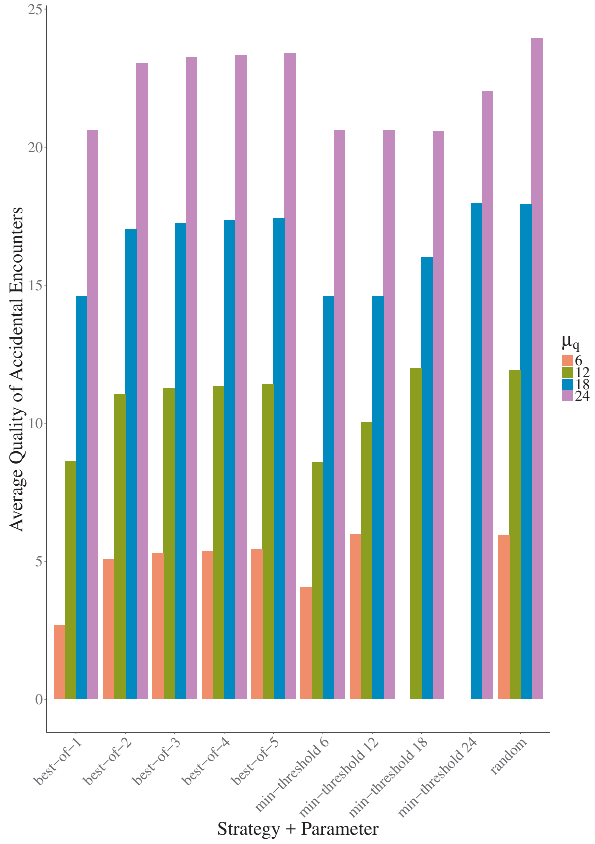


Figure 11. Effects of different selection strategies on the quality of accidental encounters for each mean quality of checkpoints. The bars represent the average quality of accidental encounters at each simulation.

hurt the performance much, because the quality of accidents is closer to the quality of selected checkpoints. Finally, when μ_q is much higher than θ, agents pursue the closest checkpoint. Therefore, accidents happen either with satellites or because a new checkpoint just appeared in front of the agent. The quality of these accidental checkpoints also does not hurt the performance much because their quality is close to the original choice of the agents.

What is the Influence of Satellites on Accidents?

Low quality checkpoints have a small chance of being selected by agents using non-random strategies. These checkpoints benefit from accidental encounters in situations where an agent moving towards a better checkpoint bumps into a low quality one. In a fully observable environment, agents can explicitly avoid low quality checkpoints. However, in a partially observable environment, agents do not know the location of the worst checkpoints. Hence, the chance of agents accidentally encountering these checkpoints will increase.

Figure 12 shows the average number of accidental encounters for different selection strategies and different types of environments. In configurations with 10% of satellites, when agents used any of the selection strategies, more checkpoints were accidentally visited.

It is also important to identify how many of these accidents happened with satellites in a partially observable environment. Figure 13 depicts the number of accidents with satellites over the total number of accidents on $\omega = partially - observable$. In our experiments with a partially observable environment, 10% of checkpoints were satellites. The results showed that in configurations with agents using either *best-of-n* with $n = 1$ or *min-threshold* with $\theta = 6$, more than 15% of accidents happened with satellites. In addition, all configurations except agents using *min-threshold* with $\theta = 24$, more than 5% of accidents happened with satellites on average.

We also compared the average quality of accidents with checkpoints not displaying their location and quality (i.e., satellites) and with checkpoints displaying their location. Looking at the results of all simulations with $\omega = partially - observable$, the average quality of accidentally visited checkpoints displaying their location was 14.570 and the average quality of visited satellites was 12.191. Table 5 shows the average quality of accidents with satellites for all strategies. We can see that increasing the parameter θ also increases the average quality of the visited satellites. This is because when agents have a high threshold and the average quality of checkpoints is low, they do not even enter the environment, hence it does not matter whether checkpoints display their location/quality or not.

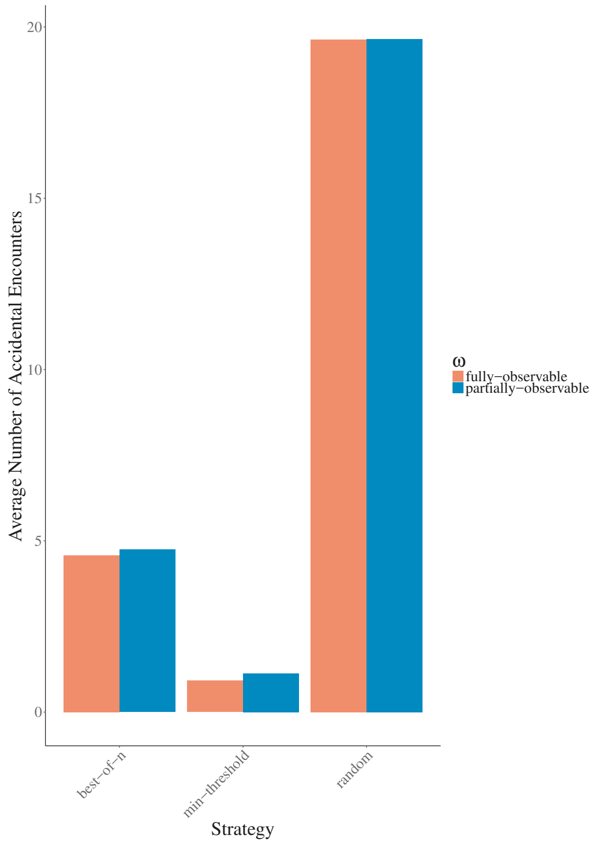


Figure 12. Effects of different selection strategies on the number of accidental encounters for each ω. The bars represent the average number of accidental encounters at each simulation.

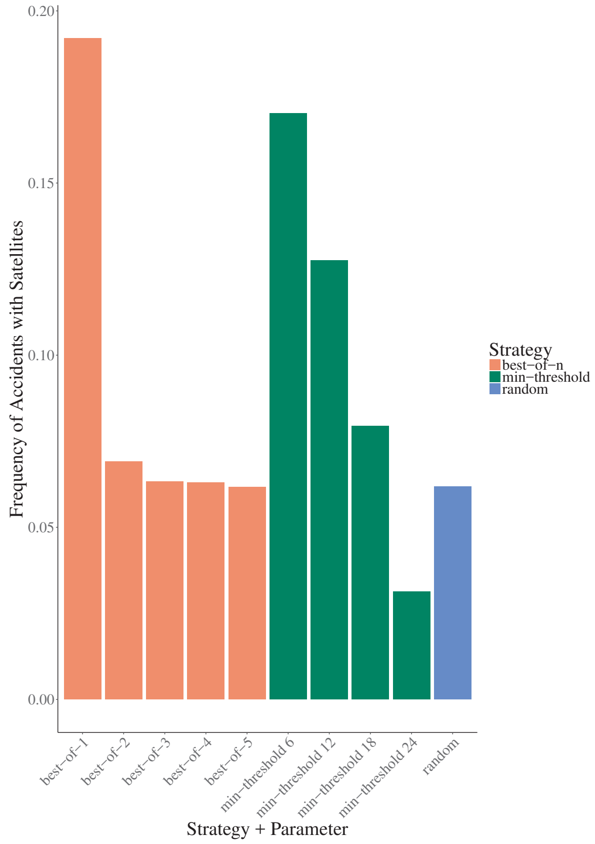


Figure 13. Effects of different strategy parameters on the frequency of accidents with satellites. The bars represent the average of the number of accidents with satellites over the total of accidents for each simulation.

Strategy	MVS
random	11.547 ± 6.709
best-of-n	11.567 ± 6.718
best-of-1	11.549 ± 6.723
best-of-2	11.558 ± 6.720
best-of-3	11.599 ± 6.712
best-of-4	11.568 ± 6.719
best-of-5	11.558 ± 6.718
minthresh	13.897 ± 6.223
minthresh 6	11.272 ± 6.791
minthresh 12	13.179 ± 5.855
minthresh 18	16.187 ± 4.147
minthresh 24	19.203 ± 2.603

Table 5. Summary of means and standard deviations for mean visited satellites (MVS) for each strategy and parameter.

Where are the Best Locations for Checkpoints to Increase their Chance of Being Visited?

The distribution of checkpoints is important for the frequency of accidents. Figure 14 shows the number of accidents for each distribution and different number of agents. The *Gaussian* distribution led to more accidents than the other two. This can be explained by the fact that in the *Gaussian* distribution, checkpoints exist in big a "cluster" in the center of the environment. Therefore, agents more frequently bump into unintended checkpoints also located in the area.

To further investigate the spatial effects of checkpoint placement, we created a "high-level" simulation that can predict, for each distribution, the location where accidents are more likely to happen. The goal of the high-level simulation is to find checkpoint positions with high chances of being visited by accident.

Algorithm 1 shows the main procedure of the high-level simulator. It starts dividing the 20m X 25m environment into 2000 50 cm^2 square sites. Then, 100 checkpoints are placed in sites according to the three distributions (*Gaussian*, *inverseGaussian* or *uniform*). For each site in the $sitesList$, one agent is placed in that site and the closest checkpoints to that agent are stored in the $closestCheckpoints$ list. The size of this list depends on the strategy used by the agent. When $a_{\pi(p)} = random$, $closestCheckpoints$ contains all checkpoints in the environment. When $a_{\pi(p)} = best - of - n$, $closestCheckpoints$ contains n elements. When $a_{\pi(p)} = min - threshold$, $closestCheckpoints$ contains 50 elements because we assume that half of the checkpoints have quality above θ.

In the high-level simulation, it does not matter the actual quality of a checkpoint, because the high-level simulation tests all paths from the agent to the $closestCheckpoints$, and check whether an accident occurs. For all checkpoints in $closestCheckpoints$, we trace a line from the agent's site to the checkpoint. This line represents a possible trajectory

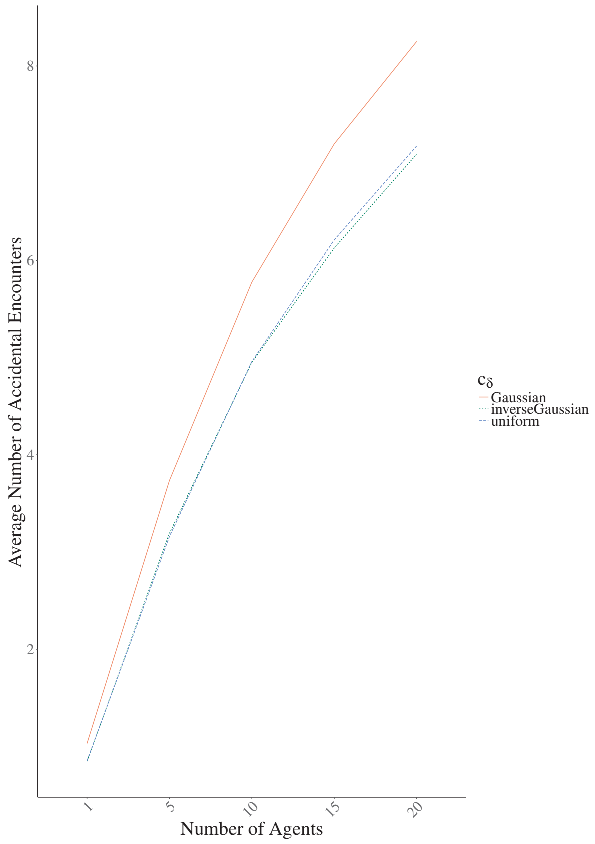


Figure 14. Interaction between the distributions of checkpoints and the number of agents on the number of accidents.

from the agent until it reaches the checkpoint. When this trajectory intercepts a different agent, an accident occurs.

Let the checkpoint $C(i)$ be the i^{th} closest to the agent, i.e., there are $i-1$ checkpoints closer to the agent than $C(i)$. Also let $Q(C(i))$ be the quality of checkpoint $C(i)$. For *best-of-n*, $\forall C(i) \in sitesList, i \leq n$, the probability that the agent selects $C(i)$ is equal to $\frac{1}{n}$. In this case, all checkpoints have the same probability of having the best quality. However, for *min-threshold* case, this property does not hold true. For *min-threshold*, the probability that an agent selects a checkpoint $C(i)$ is the probability of $Q(C(i)) \geq \theta$ and $\forall C(j) \in sitesList, \{j \leq i \text{ and } Q(C(j)) < \theta\}$. Thus, $addAccident$ adds the probability of this checkpoint being selected.

Figure 15 shows the heatmaps generated from the high-level simulation. Each square shows the probability of an accident from an agent starting in that site. As there is no accident when agents used the *best-of-n* strategy with $n = 1$ (Ferreira & Scheutz, 2015), we did not add this configuration to Figure 15.

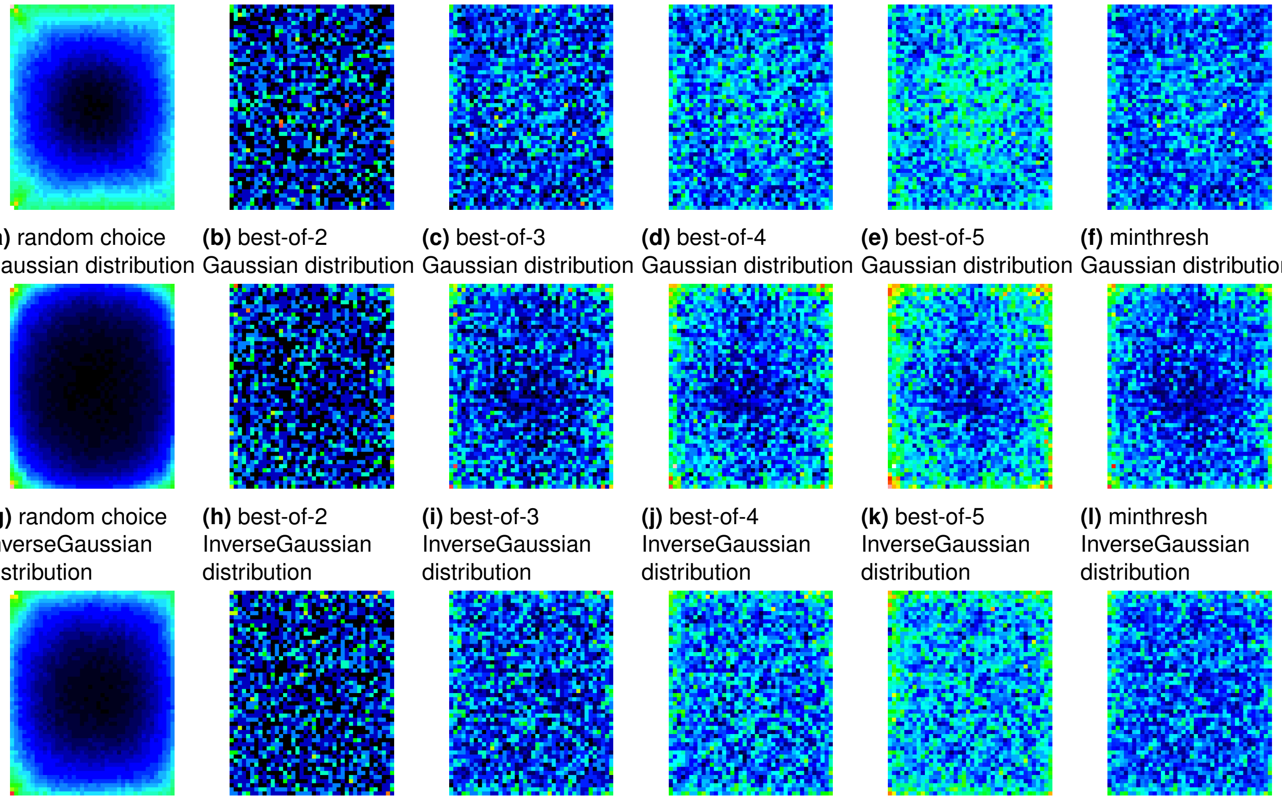
Looking at heatmaps for each strategy, they showed interesting patterns of accidents. Results from the *random* strategy display a circular shape where agents located in areas close to the edges have a higher probability of encountering an accident in their paths toward a checkpoint. In the *inverseGaussian* distribution, the vast majority of accidents occur when agents are in the corners, while in the *Gaussian* distribution more accidents occur close to the center as there are more checkpoints near the center of the environment.

Algorithm 1 Pseudo code of the process performed by the high level simulation.

HLSim.run$(seed, a_{\pi(p)}, c_\delta)$
$sitesList \Leftarrow divideEnvironment(2000, 2500, 50)$
$Agent\ a \Leftarrow newAgent(a_{\pi(p)})$
$cpList \Leftarrow createCP(seed, 100, c_\delta, 0)$
for all $Site\ s \in sitesList$ **do**
  $placeAgent(a, s)$
  $closestCheckpoints \Leftarrow getClosestCP(cpList, a, a_{\pi(p)})$
  for all $Checkpoint\ c \in closestCheckpoints$ **do**
    $line \Leftarrow traceLine(a, c)$
    $hasCrossed \Leftarrow checkLine(line, cpList)$
    if $hasCrossed == true$ **then**
      $addAccident(s, a_{\pi(p)})$
    end if
  end for
end for
$return(sitesList.accidents)$

The results of our agent-based simulation showed a direct correlation between the parameter n in the *best-of-n* strategy and the frequency of accidents. The same outcome is replicated by the high-level simulation. However, looking at the heatmaps of *best-of-n* strategy, the distribution of checkpoints determines the accident locations as the parameter n increases. More specifically, for $n = 2$, the regions of accidents are more uniformly distributed through the environment, even with the *Gaussian* and *inverseGaussian* distributions. For $n = 2$, accidents happen with the unchosen checkpoint in the set of 2-closest. Independent of the distribution, in a set of 100 checkpoints, for all squares in the environment, there are two checkpoints that are relatively close to them. Thus, accidents happen in a more or less uniform fashion. However, starting at $n = 3$, a cluster of accidents in the middle of the environment starts to emerge in the *Gaussian* distribution while accidents near the corners emerge in the *inverseGaussian* distribution. The reason is that, as n increases, the $n - closest$ checkpoint tends to belong to areas with more checkpoints. Therefore, as n increases, more accidents happen in areas with more checkpoints. Finally, in *min-threshold* the number of accidents depends on the mean quality of checkpoints. Consequently, the location of checkpoints uniformly determines the distribution of accidents. Thus, the *inverseGaussian* distribution has more accidents near the borders and the *Gaussian* distribution shows some small areas with high density of accidents in the center.

These results demonstrate that accidents happen with higher frequency close to the beginning of the simulation. More specifically, during the first cycles (i.e., as agents are still close to the edges), the chosen checkpoint is positioned inside a $\frac{\pi}{2}$ angle. In addition, when agents are located in the center of the environment, the path towards the chosen checkpoint is angled at 2π. Figure 16 shows an example of agent near the bottom edge of the environment and another



(a) random choice Gaussian distribution **(b)** best-of-2 Gaussian distribution **(c)** best-of-3 Gaussian distribution **(d)** best-of-4 Gaussian distribution **(e)** best-of-5 Gaussian distribution **(f)** minthresh Gaussian distribution

(g) random choice InverseGaussian distribution **(h)** best-of-2 InverseGaussian distribution **(i)** best-of-3 InverseGaussian distribution **(j)** best-of-4 InverseGaussian distribution **(k)** best-of-5 InverseGaussian distribution **(l)** minthresh InverseGaussian distribution

(m) random choice uniform distribution **(n)** best-of-2 uniform distribution **(o)** best-of-3 uniform distribution **(p)** best-of-4 uniform distribution **(q)** best-of-5 uniform distribution **(r)** minthresh uniform distribution

Figure 15. Heatmaps of accidental matings. Each square represents a site where an agent can exist. The colors represent the probability of an agent starting at that site accidentally visit any checkpoint in the environment, for each strategy and each distribution of checkpoints.

agent already in the middle of the environment. The number of possible paths through the environment when the agent is located near the edges is smaller than when agents are already in the middle of the environment. Therefore, there is a higher probability that accidents will happen close to the beginning of the simulation.

Finally, with *best-of-n* and *min-threshold* strategies, the probability of accidents per site for each distribution can be ordered in $gaussian < inversegaussian < uniform$. These results are different from the results of the agent-based simulation. Our hypothesis for that discrepancy is that in the high-level simulation, the chance of an agent to be in any location of the environment is the same (as we systematically calculated accidents for each location), while in the agent-based simulation some areas of the environment are more likely to have an agent than others. For example, as agents start at the borders of the environment, they spend more time in areas close to the borders than in the center of the environment. Still, the period agents spend in each area varies according to the strategy agents use and the distribution of checkpoints. In sum, the high-level simulation is important as an estimate of accidents that can give us an intuition of good and bad locations for checkpoints. However, the complex interaction between strategies and checkpoint distributions further justifies the use of an agent-based model instead of just a high-level simulation.

Discussion

In (Ferreira & Scheutz, 2015) we performed experiments in a mating task in which female frogs selected males using *best-of-n* and *min-threshold* strategies, and after mating, both frogs left the environment. In that paper, we reported that *min-threshold* performed better not only because it had a better performance than *best-of-n* in terms of average mated male quality, but also because it led to fewer accidents than the other strategy. We performed independent-samples t-tests to compare whether both strategies perform significantly different in the more general MATE task. When agents do not leave the environment, the *best-of-n* strategy led to fewer accidents than the *min-threshold* strategy $(t(832080) = 503.13,\ p - value < 2.2e-16)$. Moreover, agents playing the *best-of-n* strategy also visited more checkpoints on average than agents playing the *min-threshold* $(t(998310) = 246.25,\ p - value < 2.2e-16)$, although the latter had a better performance in terms of the quality of visited checkpoints $(t(809440) = -183.05,\ p - value < 2.2e-16)$.

A specific behavior emerged due to agent competition: Agents were attracted by the best local checkpoints according to their strategy. However, as agents remained in the environment for the entire simulation, as time passed and agents visited checkpoints, distinct agents started to have the same local best. Hence, different agents chose the same checkpoint. This continued into the next checkpoints, leading to agent groups that followed the same path. The

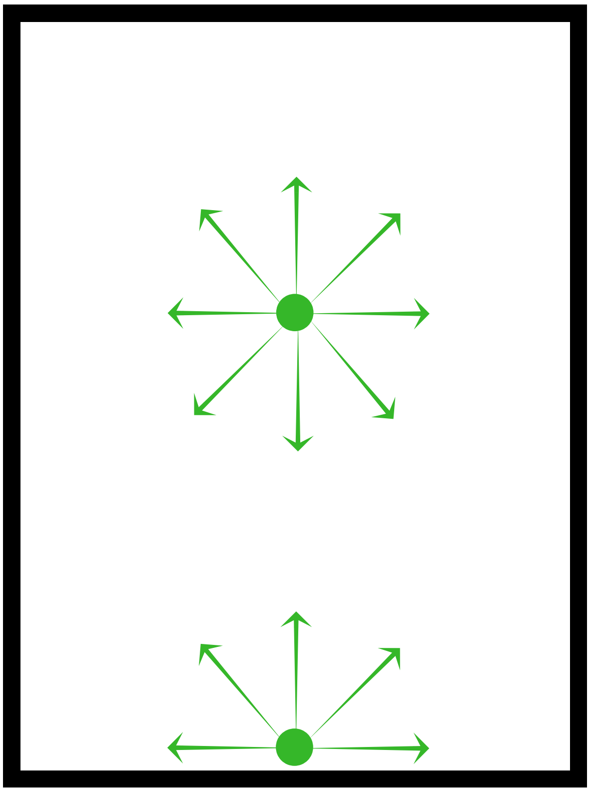


Figure 16. Environment with two agents. Agent close to the edges (i.e., start of the simulation) has fewer possible paths than another agent that is in the middle of the environment which can move to any direction.

consequence of this behavior is that if a path would lead to an accident, independently of the number of agents in the same path, the number of accidents would always be one. While increasing the number of agents increases the probability of accidents, it also increases the chance that various agents have the same path toward checkpoints, which does not increase the frequency of accidents. The distance between any pair of checkpoints is longer, on average, in the *inverseGaussian* distribution. Consequently, it takes longer for agents to have the same local best. Therefore, this grouping behavior is rarer in the *inverseGaussian* distribution.

In order to compare the performance of agents in a MATE task, it is necessary to define which performance metric is more relevant to agents. For example, if one is interested in visiting most checkpoints within a given time, then our results show that agents using the *best-of-1* strategy (i.e., always move toward the closest checkpoint) is the best approach (the same outcome can be achieved with a *min-threshold* strategy where the threshold is lower than the quality of any of the checkpoints). Moreover, if agents are aware of all checkpoints from the start, *best-of-1* eliminates accidents and reduces competition among agents. However, if the goal is to have a high quality of visited checkpoints, it is important to avoid lower quality checkpoints. *min-threshold* with a medium quality threshold can avoid these undesirable checkpoints. Although agents using this strategy had a higher frequency of accidents than the *best-of-n* strategy, the quality of visited checkpoints is much higher than any parameter assignment for the *best-of-n* strategy.

There are three types of accidents: accidents with visible low quality checkpoints (Type 1), and accidents with invisible "satellites" (Type 2 and Type 3). Accident of Type 1 happens when an agent senses and ignores a "bad" checkpoint (i.e., full observability), but the checkpoint is directly in the agent's path to another selected checkpoint. On the other hand, the agent could not avoid an encounter with a checkpoint that was just spawned in the agent's path or a checkpoint does not advertise its location (i.e., partial observability), using a better navigation strategy, say, as would be possible in the case of visible low quality checkpoints.

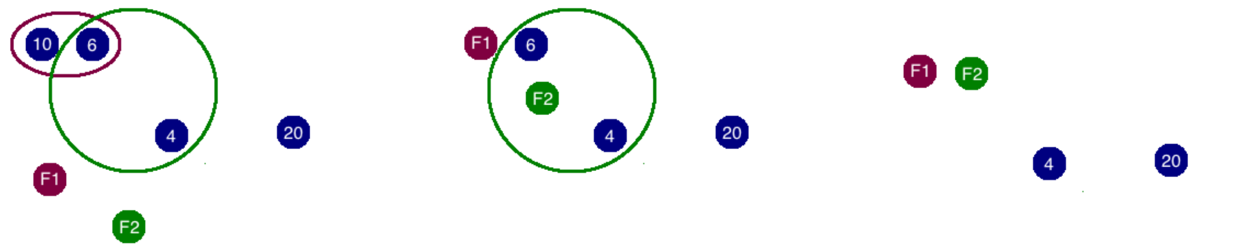
It is interesting to note that accidents in both cases of visible and invisible checkpoints are not always bad for the whole population. Looking at cases where accidents happen with low quality checkpoints, we could see some configurations in which accidents increase the average quality of visited checkpoints. For example, Figure 17 and Figure 18 show two similar environments that demonstrate how accidents can be advantageous for the whole population of agents. These two figures show two agents ($F1$ and $F2$) and various checkpoints with different qualities, with higher numbers representing better checkpoints. In these examples, agents are following the *best-of-2* strategy. Hence, they sample the closest two checkpoints and decide to move towards the best between those two. Moreover, for the sake of simplicity, we will assume that agents stop after visiting a checkpoint.

Figure 17a shows the start of the simulation. Agent $F1$ samples the checkpoints 10 and 6 and agent $F2$ samples the checkpoints 6 and 4. At the moment of Figure 17b, $F1$ reaches the location of checkpoint 10 while $F2$ continues the movement towards checkpoint 6. The simulation ends with $F2$ reaching checkpoint 6. The average quality of visited checkpoints in this example is 8.

Figure 18a shows a similar start configuration with the only change being a new checkpoint 2 close to $F1$. In this case, $F1$ samples the checkpoints 10 and 2, while $F2$ samples checkpoints 2 and 4. $F1$ moves toward checkpoint 10, however, at Figure 18b, $F1$ accidentally encounter checkpoint 2. This leads $F2$ to change its choice to checkpoint 20 (best between 4 and 20). At the end of the simulation, $F2$ reaches checkpoint 20. In this simulation, even though there is an accident, the average quality of visited checkpoints is better than that in Figure 17 (11 against 8). This is an example of how accidents can be beneficial to the average quality of the encounters.

Agents in the previous examples use *best-of-2* as selection strategy. The same behavior might exist in other values of n. However, one might wonder whether agents using *min-threshold* display accidents that might increase the average fitness of the population. This type of accident does not happen with agents following *min-threshold* because agents always move towards the closest checkpoint with quality above θ. Therefore, agents only select a different checkpoint when another agent visited their original choice.

Assume agent a_1 is moving towards c_{high} but accidentally visited checkpoint c_{low}. Agent a_2 has the same θ as a_1. As a result, it is impossible for a_2 to select c_{low} on purpose.

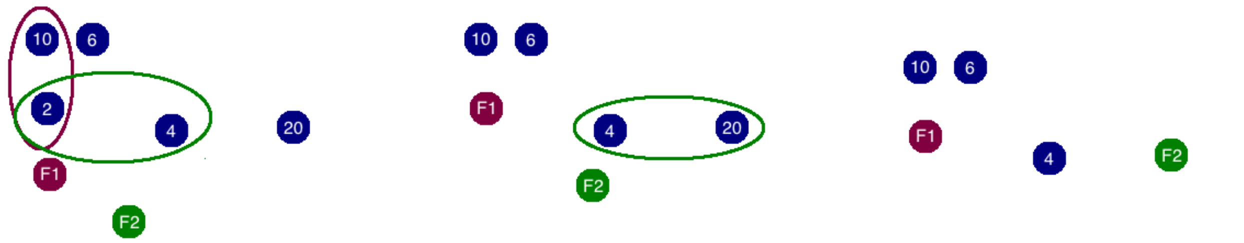


(a) $F1$ samples checkpoints 10 and 6 and starts moving towards 10. $F2$ samples checkpoints 4 and 6 and starts moving towards 6.

(b) $F1$ visits checkpoint 10. $F2$ maintains your original sample subset to checkpoints 4 and 6 and it continues its trajectory towards 6.

(c) $F2$ visits checkpoint 6. Final average quality is equal to 8 ($\frac{10+6}{2}$).

Figure 17. Environment configuration with two agents ($F1$ and $F2$) following the *best-of-2* strategy and four checkpoints (blue circles) with distinct qualities (numbers inside the circles). The blue circles inside the red ellipse are the checkpoints sampled by $F1$ and the blue circles inside the green ellipse are the checkpoints sampled by $F2$. 17a shows a possible initial condition. 17b shows the environment at the moment that $F1$ visits the checkpoint with quality 10. And 17c shows the final configuration at the moment that $F2$ visits the checkpoint with quality 6.



(a) $F1$ samples checkpoints 10 and 2 and starts moving towards 10. $F2$ samples checkpoints 2 and 4 and starts moving towards 4.

(b) $F1$ accidentally visits checkpoint 2. Thus, $F2$ changes its sample subset to checkpoints 4 and 20 and starts moving towards 20.

(c) $F2$ visits checkpoint 20. Final average quality is equal to 11 ($\frac{2+20}{2}$).

Figure 18. Environment configuration with two agents ($F1$ and $F2$) following the *best-of-2* strategy and five checkpoints (blue circles) with distinct qualities (numbers inside the circles). The blue circles inside the red ellipse are the checkpoints sampled by $F1$ and the blue circles inside the green ellipse are the checkpoints sampled by $F2$. 18a shows a possible initial condition. 18b shows the environment at the moment that $F1$ accidentally visits the checkpoint with quality 2. And 18c shows the final configuration at the moment that $F2$ visits the checkpoint with quality 20.

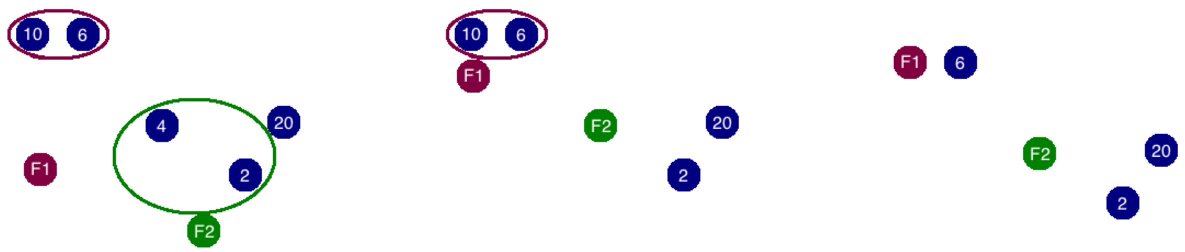
Hence, a_2 does not change its original choice. We know that c_{low} has quality below c_{high}, otherwise a_1 would have selected c_{low}. Therefore, accidents in *min-threshold* always reduce the average quality of the population in a fully-observable environment.

Type 3 accidents happen between an agent and a "satellite" checkpoint. A satellite is a checkpoint that does not advertise its quality. For example, the mating task in some species of frogs contains some males that do not call, hence, they are not sensed by females (because they do not call and the females cannot see them at night). Instead, they position themselves near "good callers" and intercept females approaching the calling site. Thus, females use strategies to avoid bad callers, however, they cannot avoid males that they can not sense.

In our experiments, we saw a small increase in the number of accidents in partially-observable environments. In a biological mating task, the worst 10% of male mates are unlike to be selected by female agents, even though they might be important to maintain a diversity in the gene pool. For those males to have a chance at mating, they would need to adapt their strategy and avoid calling to increase their possibility of being intercepted by a female. Satellite behavior is more advantageous at high densities of males, which could be at the beginning of the breeding season, when the majority of females has not mated. Males may thus have to perform different strategies according to their own call quality and the probability of an accidental mating.

Figure 19 and Figure 20 show an example of the advantage of checkpoints becoming satellites. Figure 19a shows the start of the simulation. Agent $F1$ samples the checkpoints 10 and 6 and agent $F2$ samples the checkpoints 4 and 2. $F2$ reaches checkpoint 4 at Figure 19b, and $F1$ reaches checkpoint 10 at Figure 19c. Figure 20 depicts the same environment, but checkpoint 2 (the worst checkpoint in the environment) becomes a satellite. In this case, $F2$ samples checkpoints 4 and 20. In the path towards checkpoint 20, $F2$ accidentally encounter the satellite 2. This is the only way for checkpoint 2 to be visited by an agent. Therefore, it is advantageous for this checkpoint to become a satellite.

Accidents may happen in foraging, mating, predator-prey tasks among others. In this paper, we did not plan to have a biologically plausible simulation for a specific task, yet, we proposed a general instance of the MATE task in order for the conclusions drawn here could provide explanations of accidents in these different natural tasks. For example,

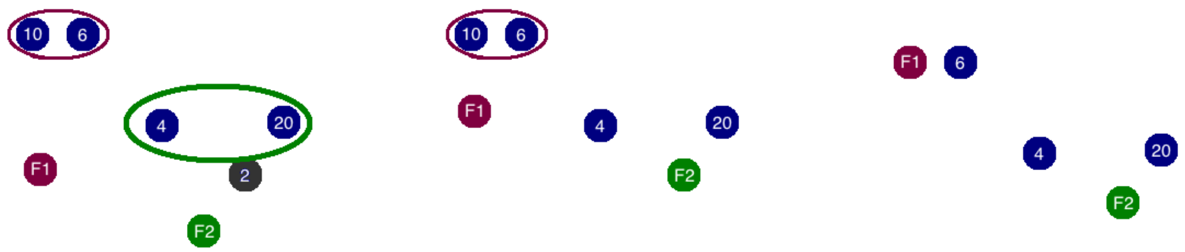


(a) $F1$ samples checkpoints 10 and 6 and starts moving towards 10. $F2$ samples checkpoints 4 and 2 and starts moving towards 4.

(b) $F2$ visits checkpoint 4. $F1$ maintains your original sample subset to checkpoints 10 and 6 and it continues its trajectory towards 10.

(c) $F1$ visits checkpoint 10. Final average quality is equal to 7 ($\frac{10+4}{2}$).

Figure 19. Environment configuration with two agents ($F1$ and $F2$) playing the *best-of-2* strategy and five checkpoints (blue circles) with distinct qualities (numbers inside the circles). The blue circles inside the red ellipse are the checkpoints sampled by $F1$ and the blue circles inside the green ellipse are the checkpoints sampled by $F2$. 19a shows a possible initial condition. 19b shows the environment at the moment that $F2$ visits the checkpoint with quality 4. And 19c shows the final configuration at the moment that $F1$ visits the checkpoint with quality 10.



(a) $F1$ samples checkpoints 10 and 6 and starts moving towards 10. $F2$ samples checkpoints 4 and 20 because checkpoint 2 is a satellite.

(b) $F2$ accidentally visits satellite 2. $F1$ maintains your original sample subset to checkpoints 10 and 6 and it continues its trajectory towards 10.

(c) $F1$ visits checkpoint 10. Final average quality is equal to 6 ($\frac{10+2}{2}$).

Figure 20. Environment configuration with two agents ($F1$ and $F2$) playing the *best-of-2* strategy and four checkpoints (blue circles) and one satellite (gray circle) with distinct qualities (numbers inside the circles). The blue circles inside the red ellipse are the checkpoints sampled by $F1$ and the blue circles inside the green ellipse are the checkpoints sampled by $F2$. $F2$ can not sense the satellite. 20a shows a possible initial condition. 20b shows the environment at the moment that $F2$ accidentally visits the satellite with quality 2. And 20c shows the final configuration at the moment that $F1$ visits the checkpoint with quality 10.

while an accident that leads to an interspecies mating may bring the female's breeding season to an end, the same accident when led to a intraspecies mating may be beneficial to maintain the diversity in the gene pool. Looking at a foraging task, a food source that would only be gathered by accident would benefit of a high probability of accidents, a hidden predator would also benefit from staying on areas where the likelihood of accidents is higher. However, when the animal can only gather a small amount of food, accidents would damage its performance. Therefore, in order to draw conclusions of particular instances of the MATE task, a particular simulation must be implemented and then the results may be compared with our findings.

Conclusion

Accidental encounters typically decrease performance of agents in multi-agent territory exploration (MATE) tasks. In this paper we investigated the frequency of accidental encounters and the influence of those accidents on the overall performance of an agent group in an instance of the MATE task. After running a large parameter sweep in which we varied the type of environment (partially-observable vs. fully-observable), ratio of agents and their strategies, as well as the mean quality, distribution and method of placing checkpoints, we showed that using non-random checkpoint selection strategies overall decreased the frequency of accidents compared to random checkpoint selection.

There are, however, cases in which accidents are beneficial for the agent and the agent population as a whole. One example is a situation where a new high-quality checkpoint appears near an agent before the agent could sense it, which can increase the average performance of the population and also lead agents to visit those checkpoints right away. We also showed a more complex scenario in which an accident led agents to select a better checkpoint that would not have been selected without the accident.

In biological settings, accidents might ensure the diversity of the gene pool and thus be adaptive. I.e., without ways for low quality males to mate, the diversity of the gene pool would shrink over time, thus potentially leading to too much specialization with all of its consequence (e.g., lack of adaptability to environmental changes).

We also found an emergent behavior that happens in MATE tasks where agents are allowed to visit any number of checkpoints. In those cases, agents tend to group themselves and move together to visit local best checkpoints. This emergent behavior reduces the quantity of visited checkpoints as well as the number of accidents, because agents wander through the same path, therefore a large group can visit only one checkpoint at a time. This behavior might be beneficial for satellites and assist them in finding a good location where they increase the likelihood of accidental mating. Because more agents would be approaching local bests, there is a higher chance that the satellites would intercept some of them.

While in the instance of the MATE task we investigated in this paper checkpoints remained stationary, it is useful to consider instances where they can move as well. Evidence from preliminary simulations shows that allowing checkpoints to place themselves based on other checkpoints' placements move improves the average performance of the population (see (Scheutz, Smiley & Boyd, 2013)). It remains unclear, however, whether this improvement in performance happens because of accidental encounters. We hypothesize that the strategy used by checkpoints is determinant on the frequency of accidents. Therefore, understanding the accidents in the view of checkpoints, (i.e., what is a good strategy to increase a checkpoint's chance of being selected) would be an interesting direction for future work on accidental encounters and their potential adaptiveness for biological agents.

There are instances of MATE tasks where more or less accidents are desirable. Using the data we present in this paper, we plan to suggest changes on selection strategies to account for accidental encounters. These altered selection strategies may generate novel hypothesis about biological instances of the MATE task.

Declaration of conflicting interests

The Authors declare that there is no conflict of interest.

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Appendix A: Model Description

In this section, we present the model description according to the ODD (Overview, Design concepts, Details) protocol (Grimm, Berger, DeAngelis, Polhill, Giske & Railsback, 2010).

Purpose

The purpose of this model is to investigate the conditions that increase the probabilities of accidental encounters in a MATE task and the effects they have on the average task performance. In our experiments, we report the *frequency of accidents* and *average quality of accidents* and we compare these values to the frequency and average quality of non-accidentally visited checkpoints.

Entities, state variables and scales

Our model runs as a discrete-time simulation with a minimum time measure called (simulation) "cycle". Two entities exist in our simulations: agents and checkpoints. Agents move in a two-dimensional environment visiting checkpoints. On the other hand, checkpoints remain in the same place during the entire simulation. All entities have a unique identification name a_{id}, and their position in the environment (a_x, a_y). Agents also have a fixed velocity a_v and an orientation a_α. Agents distinguish checkpoints by the checkpoints' quality a_q.

Agents select checkpoints based on a strategy $a_{\pi(p)}$. The selection strategy π may be either the $best-of-n$ or $min-threshold$. The strategy parameter p may be assigned either to n for specific size of the set of closest checkpoints or θ for the particular minimum threshold, respectively. In addition, a random choice strategy serves as a baseline. Each agent stores in a_{chosen} the identification name of the selected checkpoint.

The environment has dimensions equal to E_x and E_y and the simulation finishes after $Term$ cycles. The quality of each checkpoint is defined by a Gaussian distribution with mean μ_q and standard deviation σ_q. An agent m will always visit a checkpoint c if their distance is smaller than $d_{visiting}$ (i.e., $d_{(m,c)} \leq d_{visiting}$).

Checkpoints might display their quality or remain "hidden". The checkpoints that do not show their qualities are called "satellites". The variable ω defines whether there are satellites in the environment. If all checkpoints display their quality, $\omega = fully-observable$. Otherwise, $\omega = partially-observable$.

All m_n of agents are positioned on the edges of the environment through a distribution m_δ. All c_n checkpoints are placed inside the environment according to a distribution c_δ and a placement method c_p. All checkpoints may be placed in the environment at the start of the simulation ($c_p = simultaneous$) or a new checkpoint may be placed at each $\frac{Term}{c_n}$ cycle ($c_p = progressive$).

Process overview and scheduling

The scheduler controls the simulation, creating agents, placing checkpoints, and counting the number of cycles until the end of the simulation. The scheduler starts by creating agents and storing them in a list. It also places agents according to m_δ. Next, the scheduler creates all checkpoints and stores them in a list, noting whether agents are satellites according to ω. The list $checkpointList$ contains c_n elements and each element has an associated quality defined by μ_q and σ_q. The next step is to create the list $availableCP$ which consists of all checkpoints that exist at every cycle. If $c_p = simultaneous$, then

$availableCP$ contains all elements in $checkpointList$. Otherwise, $availableCP$ contains only one element.

The update phase happens until the number of cycles reaches the value defined by $Term$. First, the scheduler verifies whether it is time for placing a new checkpoint, in which case a new element is added to $availableCP$. Next, the sensing phase starts. First, agents verify whether they reached a checkpoint location. Then agents decide the next checkpoint to move towards based on their strategy. Finally, the acting phase starts and then all agents move one step towards the selected checkpoint.

Algorithm 2 Pseudo code of the process performed by the scheduler.

Sim.run$(seed, a_{\pi(p)}, m_n, m_\delta, c_n, c_\delta, \mu_q, c_p, \omega, Term)$

$agentList \Leftarrow createAgents(seed, a_{\pi(p)}, m_n, m_\delta)$
$checkpointList \Leftarrow createCP(seed, c_n, c_\delta, \mu_q, \omega)$
$addCP(availableCP, checkpointList, c_p)$
$cycle \Leftarrow 0$
while $cycle < Term$ **do**
  if $c_p == 1$ **then**
    if $cycle == \frac{Term}{c_n}$ **then**
      $addCP(availableCP, checkpointList, c_p)$
    end if
  end if
  for all $Agent\ m \in agentList$ **do**
    $m.sense(availableCP)$
  end for
  for all $Agent\ m \in agentList$ **do**
    $m.act()$
  end for
end while
$return(visited, \frac{visitedSum}{visited}, accidents, \frac{accidentsSum}{accidents})$

Submodels

Sensing process At each cycle, agents start looking into the list of checkpoints to decide which checkpoint to visit (see Algorithm 3). During this process, agents first iterate through $availableCP$ testing whether the checkpoint is in reachable distance. If the agent reaches the checkpoint, the checkpoint is removed from $availableCP$ and the number and quality of visited checkpoints (accidentally or not) is stored.

Algorithm 3 Pseudo code of the sensing process performed by agent i.

sense$(i, availableCP)$

for all $Checkpoint\ c \in availableCP$ **do**
  if $d(i, c) \leq d_{visiting}$ **then**
    if $i_{chosen} \neq c_{id}$ **then**
      $accidents \Leftarrow accidents + 1$
      $accidentsSum \Leftarrow accidentsSum + c_q$
    end if
    $visited \Leftarrow visited + 1$
    $visitedSum \Leftarrow visitedSum + c_q$
    $availableCP.remove(c)$
  end if
end for
$i_{chosen} \Leftarrow selectCheckpoint(availableCP)$

At the second part of the sensing process, all agents run Algorithm 4 to select the next checkpoint to move towards. If the agent i uses the *best-of-n* strategy, then first the scheduler creates, in linear time, a list of n-closest checkpoints that are not "satellites" (i.e., n-closest that display their qualities). Afterwards, the agent chooses, in constant time, the checkpoint with the highest quality. If the agent i uses the *min-threshold* strategy, the scheduler iterates through $availableCP$ and creates a temporary list of checkpoints that are not satellites and have quality above θ. This first process takes linear time. Then agents iterate through this temporary list of checkpoints and select the closest one to them. This second process also takes linear time in the worst case (i.e., when all checkpoints in the environment have quality above θ). Finally, the random choice strategy performed by agents takes constant time.

Algorithm 4 Pseudo code of the selecting process performed by agent i.

selectCheckpoint$(i, availableCP)$

$chosen \Leftarrow null$
if $i_{\pi(p)} == best - of - n$ **then**
  $nClosest \Leftarrow chooseNClosest(i, availableCP)$
  $chosen \Leftarrow chooseBest(nClosest)$
else if $i_{\pi(p)} == min - threshold$ **then**
  $aboveT \Leftarrow selectAboveTheta(i, availableCP)$
  $chosen \Leftarrow chooseClosest(i, aboveT)$
else if $i_{\pi(p)} == random$ **then**
  $chosen \Leftarrow selectRandom(availableCP)$
end if
$return(chosen)$

Acting process During the acting process, agents perform two mathematical operations: change their heading toward the chosen checkpoint and then perform one step in that direction (see Algorithm 5 which takes constant time).

Algorithm 5 Pseudo code of the acting process performed by agent i.

act(i)

if $i_{chosen} \neq null$ **then**
  $c \Leftarrow i_{chosen}$
  $i_\alpha \Leftarrow atan(\frac{c_y - i_y}{c_x - i_x})$
  $i_x \Leftarrow i_x + (i_v * cos(i_\alpha))$
  $i_y \Leftarrow i_y + (i_v * sin(i_\alpha))$
end if