

A Scalable, Robust, Ultra-Low Complexity Agent Swarm for Area Coverage and Interception Tasks

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Abstract—Simulations of biologically inspired swarms where agents jointly achieve tasks using local rules rather than global centralized or distributed control have demonstrated the high performance of agent swarms on a variety of tasks (such as surveillance, plume tracking, or target interception). However, most swarm systems rely on the information exchange of agents with their neighbors, which in practical instantiations would involve digital communication. Moreover, many systems would require global positioning methods (e.g., GPS) to determine the exact location of agents in their environment.

We propose a beacon-based principle for target-oriented navigation of large numbers of autonomous agents, which is radically different from previous methods in that it neither requires digital communication nor any kind of global position information for coordination of movements and interactions and, moreover, has only minimal “computing” requirements. Results from extensive simulations of the system in an area coverage and agent interception task show that (1) the system achieves perfect task performance (i.e., all hostile agents are intercepted), (2) scales (works with an arbitrary number of agents), and (3) is robust (adapts to changes in agent position and configuration).

I. INTRODUCTION

Over the last few years, *swarms intelligence* [3] has become an interesting alternative to standard centralized and distributed control approaches (e.g., [22], [23], [24], [25], [26], [27]) for solving a variety of multi-agent coordination problems: This work ranges from using only local rules (e.g., [11]), “digital pheromones” [2], [4], sensor fusion [9] and the self-deployment of sensors [13], to forming formation [5], [12], chemical plume detection and tracking [1], [8], and many others.

The behaviors of agents employed in a swarm system are usually governed by simple rules, which take the position of neighbors into account (e.g., variants to Reynold’s three rules “flock centering”, “obstacle avoidance”, and “velocity matching” [6]). Based on assumptions about such simple interactions, several important theoretical results have been proved recently about properties like collision avoidance or stability in swarms systems (e.g. [15], [16], [17], [18], [19]).

While it is straightforward to obtain the exact position of an agent in simulations of swarm systems (and thus properties like the distance between any two agents), in a practical settings this information requires particular sensory and functional capacities, in particular, *digital communication* (e.g., GPS on each agent and digital communication to

relay the GPS data to other agents that require them). For example, agents in a swarm system that is based on “digital pheromones” [2], [4] will need to maintain a global map of pheromone positions that is shared among all agents.

In this paper, we propose a novel, ultra-low complexity navigation and resource allocation concept for multi-agent swarms that is exclusively based on simple radio beacons carried by each mobile unit. Different from other systems, the proposed principle can be implemented with extremely simple hardware and does not require the existence of a digital communication network to perform its functions. In principle, there are two types of beacons that every unit carries: an *attractive beacon* for resource allocation (moving autonomous agents to where they are needed) and a *repulsive beacon* to control agent distribution and density in order to avoid collisions and control sampling density. In the most general framework, all entities carrying these two beacons are assumed to be mobile, even though the case of some beacons being stationary also generates interesting applications. To demonstrate the utility of the principle, we focus on an area coverage and agent interception task, in which a swarm is deployed to protect an area from hostile intruders (e.g., UAVs or missiles). Using extensive simulations we demonstrate that (1) the system functions as expected and can achieve perfect task performance (i.e., all hostile intruders are intercepted), (2) scales (works with an arbitrary number of agents), and (3) is robust (adapts to changes in agent position and configuration).

II. THE BASIC NAVIGATION CONCEPT

As mentioned, the principle is based on two types of beacons, which each agent is equipped with: a *collision avoidance beacon* *col* (to repel agents from each other and distribute them) and a *target attraction beacon* *tar* that is only activated when agents detect a target (whatever that target may be). The collision avoidance beacons are always active and each agent is equipped with a stereo antenna/receiver pair to detect collision avoidance beacons of other agents (and their direction). In the same way, each agent uses the antenna/receiver pair to detect the target attraction beacons of other agents. Note that target attraction beacons effectively translate a signal from a target (however it was detected) into a different modality (i.e., an on/off radio beacon) to extend the range of its detectability. In contrast

to target beacon receivers, the collision avoidance receivers can extract the approximate distance of the source using the received signal strength. This allows the collision avoidance algorithm to react only to agents within a certain circle of radius ρ (the “repulsion radius”) in free space. ρ effectively is an agent’s collision avoidance range and represents the distance an agent must keep between itself and its neighbors to leave enough space to turn. Therefore, ρ is dependent on the agent’s *minimal turning radius* τ , which, in turn, is typically dependent on the agent’s speed.

For the control algorithm, we define:

$$I_{y,i} = \sum_{j=1}^n A_y / (\| \underline{x}_j - \underline{x}_i \|_2^2)$$

where $I_{y,i}$ is the received power of beacon type y at agent i at location x_i with $y \in \{col, tar\}$, and A_y is the transmit power of type y beacons. (There are a total of n agents, and all beacons of the same type have the same power.) Using the directional sensitivity of two sideway looking directional antennas, we can find the following signal intensity for left and right looking antennas of each of the two modalities:

$$R_{y,i} = \sum_{j \in \Gamma_y} A_y f(\underline{x}_j - \underline{x}_i, \underline{\eta}_i) / (\| \underline{x}_j - \underline{x}_i \|_2^2)$$

$$L_{y,i} = \sum_{j \in \Gamma_y} A_y f(\underline{x}_j - \underline{x}_i, -\underline{\eta}_i) / (\| \underline{x}_j - \underline{x}_i \|_2^2)$$

with $\Gamma_{col} = \{j \mid \| \underline{x}_j - \underline{x}_i \|_2 < \alpha\}$, $\Gamma_{tar} = \{1, \dots, n\}$, $\underline{\eta}_i$ being the right normal vector to the speed vector of agent i in the plane of operation (i.e., either on the ground or in the flight plane), and $f(x, \eta)$ being the directional sensitivity function of the antenna, where x is the vector from receiver to source and $\underline{\eta}$ is the direction of highest sensitivity of the antenna.

In the case of $y = col$, the summation for the left and right antenna signal intensity L_i and R_i respectively are taken only over those agents j that satisfy $\| \underline{x}_j - \underline{x}_i \|_2 < \alpha$. This requires certain provisions in the modulation scheme that allow the agent to distinguish each collision beacon. (In the attraction beacon receivers, distinguishing between different agent beacons is not necessary and the above summation can actually be done by the antenna itself rather than in digital hardware.)

The decision for the turn direction requires two directional antennas on each side of the agent facing in opposite directions ($\underline{\eta}$, and $-\underline{\eta}$), perpendicular to the agent speed vector. Since the turning radius of the agent is assumed independent of the direction (left and right), a simple intensity comparison between left and right directional antenna will allow to derive the new heading of the agents, which is either “turn left” or “turn right”. Define the intensity sum and difference between the antenna pairs as: $L_{y,i} + R_{y,i} = S_{y,i}$, $L_{y,i} - R_{y,i} = D_{y,i}$, $y \in \{col, tar\}$, $i = 1, \dots, n$, where $S_{y,i}$ and $D_{y,i}$ denote sum and difference of left and right antenna signal strength of modality y at agent i .

The agent guidance control algorithm is then given by:

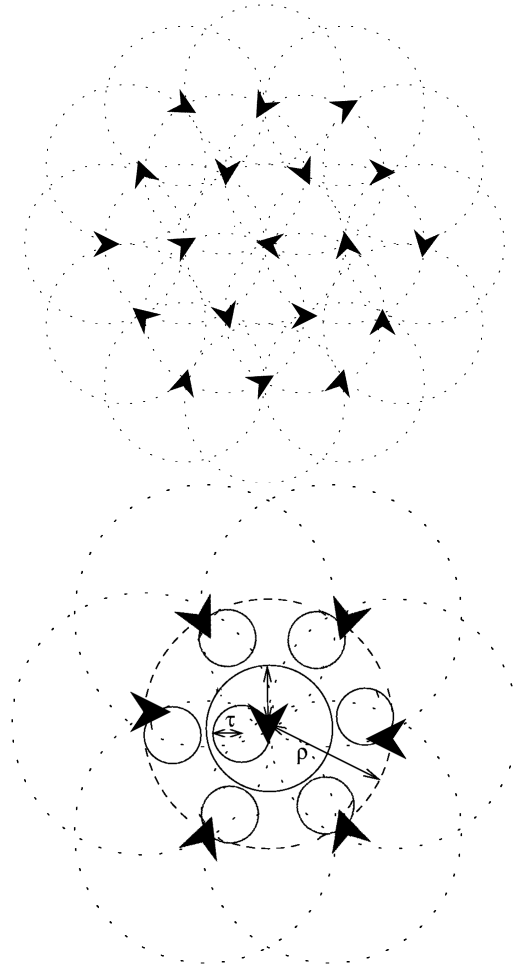


Fig. 1. Coverage of area by agents (left) and collision-avoidance (right). Dashed circles depict the repulsion range ρ . τ is the minimal turning radius.

if $S_{col,i} > 0$ & $D_{col,i} > 0$ **then** *turn right*
elseif $S_{col,i} > 0$ & $D_{col,i} \leq 0$ **then** *turn left*
elseif $S_{tar,i} > 0$ & $D_{tar,i} > 0$ **then** *turn left*
elseif $S_{tar,i} > 0$ & $D_{tar,i} \leq 0$ **then** *turn right*
else *fly straight*

On the hardware side, the navigational principle in its most simple form requires one (mobile or stationary) beacon (electromagnetic, IR, acoustic, light, etc.) and one mobile agent that has two (left, right) receivers with opposing directions of highest sensitivity that can receive the beacon and derive the received signal strength. Then using a simple comparison of the intensity of the two received signals (left versus right receiver), simple navigational decisions can be made that allow the unit to either approach or move away from the beacon. In essence, navigational decisions are based on a sequence of binary “turn left/turn right” decisions that are entirely driven by the two receiver signals.¹

¹We have shown elsewhere [1] how the algorithm can be directly realized in analog hardware without the need to use any kind of digital processor.

III. EMERGENT SYSTEM PROPERTIES

For efficient sensing and sampling, agents (e.g., UGVs or UAVs at the same altitude) must form a dense cover of subregions of the 2D plane, the density of which is determined by the agents' *repulsion range* ρ (left in Fig. 1; the dashed circles depict this radius ρ). Given one attractive beacon in the center of an area, for example, agents will automatically arrange (and continuously re-arrange) themselves in the vicinity of the beacon such that agents are outside each other's repulsion ranges. Specifically, since agents within each other's repulsion ranges move away from each other, while being attracted to the center of the region when they are not within each other's repulsion ranges, we will get a stable oscillation (i.e., a pattern of agents moving in and out of each other's repulsion ranges while staying in the same overall area). Simulations demonstrate that this behavior of agents leads to an emergent arrangement of agents on a hexagonal grid of approximately ρ grid length (i.e., the shortest distance between two agents before they ignore the attractive beacon and turn away from each other based on their navigation control system). Note that this is the tightest possible packing of circles in the plane, and thus the optimal arrangement of agents with circular non-overlapping repulsion regions.

We also conjecture that if repulsion ranges are chosen carefully such that the minimal turning radius $\tau < \rho/4 - \delta$, where δ is some safety distance, then it is always possible for agents to avoid collisions.² In the worst case, they will be able to repeat a circular pattern of radius τ in a region within their repulsion range ρ . Specifically, as shown on the bottom in Fig. 1, a complete enclosed agent can still safely turn away from a set of six surrounding agents, all of which have penetrated the repulsion range of the enclosed agent (note that these agents also have a safe place to turn within the enclosed agent's repulsion range).

One of the most important properties of the employed principle, which set it apart from other, especially centralized control approaches, is that systems based on it "scale", i.e., new agents can be simply added to a system without negatively impacting the performance of other agents. For example, if a system S consisting of k agents has achieved insufficient coverage of an area A (i.e., only $p \cdot A$ for $0 < p < 1$ is covered), then perfect coverage of A can be achieved by adding at least $(1 - p) \cdot A/2\pi \cdot \rho^2$ new agents of the same type.

Similarly, if a system S consisting of k agents has achieved coverage of an area A by covering $A + d$ (where d is the excess area covered), then removing an agent (e.g., because it ran out of fuel or was destroyed) will either still cover A (if $d > 2\pi \cdot \rho^2$, i.e., the area covered by one agent) or A will occasionally not be entirely covered.

²While we have recently made progress on the theoretical side, we have not yet been able to firmly establish collision-free navigation, hence we refer to it as a "conjecture", the main problem with a rigorous formal argument being that some basic properties of the proposed system (e.g., that UAVs fly at a constant minimal speed) make it very difficult to apply standard techniques for establishing collision-free navigation; especially those based on potential fields are either not directly applicable or difficult to apply (e.g., compare to [15], [16], [17], [18], [19], [20]).

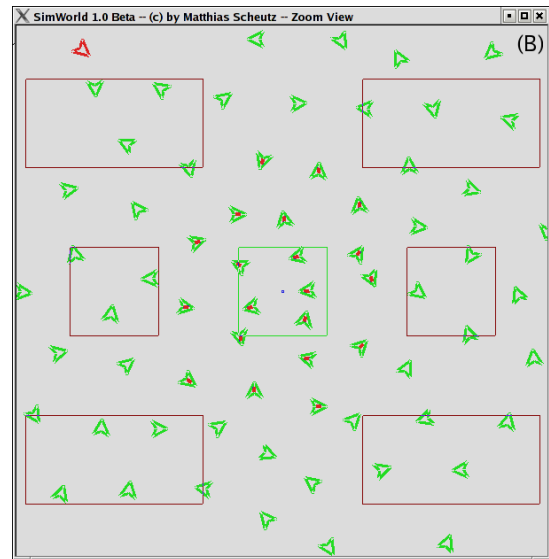


Fig. 2. The area coverage and interception task (see text for explanation).

IV. THE AREA COVERAGE AND INTERCEPTION TASK

The task is to protect a target area with well defined dimensions from incoming intruders (e.g., hostile fighters, UAVs, rocket-propelled granates, missiles, etc.). The goal is to achieve a close to 100% intercept rate with a minimal number of protecting agents using the proposed beacon principle and, most importantly, *without* the need to detect incoming agents. Hence, neither sensors for detecting incoming intruders nor computational mechanisms for tracking are required. Effectively, the agents will form a *protective shield* above the target area without having to be able to detect or track incoming objects. Whenever an incoming object attempts to penetrate the shield, at least one agent will collide with it, thus destroying both, the defending agent itself and the hostile agent on impact.

A. Task Description

For the "area coverage and interception task" we assume a hostile intruder approaching from a distance r_h with constant velocity v_h from an arbitrary direction at angle θ that leads right through the target area. For the friendly agents, we assume a constant velocity v_f with "destruction range" r_f (possibly only the diameter σ of the agent), within which a collision with the friendly agents leads to a destruction of the colliding object. Additional free parameters of the task setup are the extension of the area to be protected given by the radius r_A , the number n_f of available friendly agents, and the minimum allowable destruction distance from target location r_{min} , outside of which a hostile agent must be destroyed. Dependent parameters are the repulsion radius ρ of the friendly agents and the speed ration v_f/v_h .

The general question of interest is to find the largest repulsion radius $\rho > \tau$ (the minimal turning radius) such that there is no collision-free trajectory for intruders from any position at distance r_h and relative angle θ to the target

area if friendly agents are *tightly packed* (with respect ρ) over the target area. Note that such a ρ always trivially exists, for example, if all agents could remain in place with $v_f = 0$ (e.g., in the case of blimps or helicopters), we can take $\rho = \sigma$ (i.e., the diameter of the agent).

It is useful to distinguish intruders that can only fly straight from those that can turn (at minimum turning radius τ_h , which will depend on their speed v_h). The latter are clearly more difficult to deal with, since they might be able to avoid collisions and dynamically react to changes in swarm configurations. In either case, the success of the swarm system will depend on the speed ratio v_f/v_h , the radius ρ , and the number of employed agents n_f . We are particularly interested in lower bounds on n_f that guarantee good performance of the system (i.e., close to 100% interception). Note that we make no assumptions on the navigation control algorithm used by the intruder to avoid intercepting agents. In the degenerate case of $v_f = 0$ corresponds to the problem of optimally placing “air mines” around the target so that no collision free line through the target exists. In the case of $v_f \neq 0$, the minimum speed v_f and largest repulsion radius ρ are of interest such that interceptions of intruders are guaranteed.

B. Experimental Setup and Agent Models

All simulation experiments reported in this paper were conducted in our distributed agent-based simulation environment SWAGES, which consists of the parallelizable SIM-WORLD simulator and an experiment grid-server, which can be used to schedule experiments on heterogeneous clusters of computers, automatically parallelize and distribute simulations over multiple hosts, collect statistics, and perform preliminary data analysis [32].³

In the experiments we modeled an environment, in which two types of small structures (e.g., tents) of size $5m \times 10m$ and $5m \times 5m$ were placed in a plane (see Fig. 2 from a bird’s eye perspective). The target structure itself ($5m \times 5m$), which was to be protected by the agents, was placed in the center. An attractive ground beacon was placed in the center of the target structure and could be detected by friendly agents within a radius of $15m$ (detection of the ground beacon causes agents to turn on their attractive *tar* beacon).

Friendly agents have a circular destruction range $r_f = 1m$ (i.e., a diameter of $2m$, which is the same as their size σ). Their repulsion range ρ was varied from $6m$ to $14m$ and their speed was co-varied from $3m/s$ (i.e., $10.8km/hr$) to $21m/s$ (i.e., $75.6km/hr$) to account for the relation between speed and minimum turning radius τ (on which ρ depends) in different simulations. Within each simulation, the speed was constant. The control algorithm employed in friendly agents was the one described in Section II for all simulations.

Intruders also had the same size ($2m$), but their speed was $300m/s$ (i.e., $1080km/hr$) in all simulations. Different from friendly agents, we assumed that intruders had the ability to sense all agents (e.g., via radar), and moreover, they know

at all times where the target area is (e.g., via GPS). To avoid collisions, intruders used a potential-based control, which compute a “repulsive force” for each sensed friendly agents and an attractive force for the target location, both of which drop off with the square of the distance c/r^2 , where c is a scaling constant ($c = 100$ for the target and $c = -10$ for friendly agents).

In all simulations reported here, we used $n_f = 90$ agents. Initially, all agents were placed randomly above the center area. All simulations were run either until the target was destroyed by an intruder, or, if the target was not destroyed, for up to 1000000 simulation cycles, where one cycle corresponds to $100ms$ (i.e., for a total of about $28hrs$). All results reported here are averages over 100 different initial conditions of random placements of friendly agents. After the first 500 cycles, during which the swarm system was allowed to form a tight pattern over the target structure, a new intruder was added to the simulation, regardless of whether other intruders were still present, approaching from a distance of $r_h = 100$ from the west.

C. Experiments and Results

We first conducted a set of systematic “parameter sweep” experiments to determine the relations among different parameter settings for the swarm, most importantly, the ratio v_f/v_h . The simulation setup was as described above. Note that since agents that intercept intruders and consequently get destroyed are not replaced in these simulations, hence the number of agents will continue to drop as time goes on. Eventually, there will not be enough agents left to provide sufficient coverage of the area and intruders will be able to destroy the target. Since we expect the best configurations to last longest, we can use “time to target destruction” as a performance measure of an agent swarm.

As can be seen from the results reported in Fig. 3, lower-speed systems with lower repulsion radii last much longer before the target is destroyed than higher speed systems with higher repulsion radii. This is due to the tighter coverage of the area, which makes it more difficult for intruders to navigate through friendly agents without crashing into them. The best configuration with speed $v_f = 6m/s$ (i.e., ratio $v_f/v_h = 1/50$) and repulsion range $\rho = 7.33m$ lasted for almost $8min$ on average, destroying almost 40 intruders.⁴

To determine how well systems (like the above with $v_f = 6m/s$) perform under more realistic assumptions of replenishments of destroyed agents, we conducted another set of experiments for three systems with $v_f \in \{6m/s, 7m/s, 8m/s\}$ and the corresponding values for ρ as depicted in Fig. 3). In these experiments, a new agent was added to the swarm at a random location within the area covered by the swarm for each intruder that was destroyed.

The results obtained for all three conditions are very promising: (1) no friendly agent collided with another agent

⁴Note that there was a difference between the number of destroyed intruders 48.36 and the number of destroyed agents 39.92, which is due to the fact that sometimes the same agent destroyed two intruders at the same time (see also the discussion of the second set of experiments below).

³SWAGES is freely available at <http://www.nd.edu/~airolab/software/>.

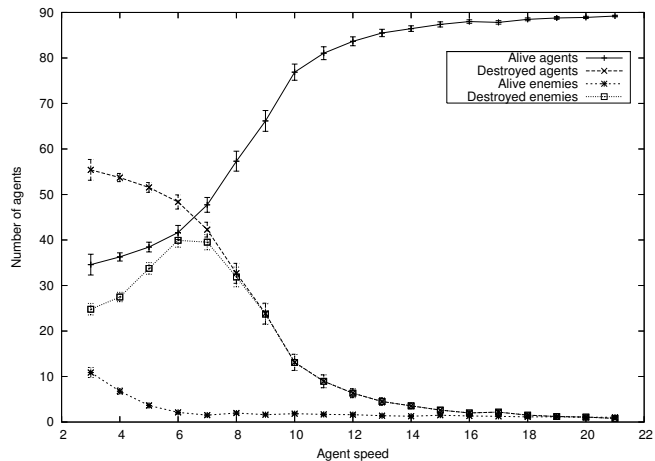
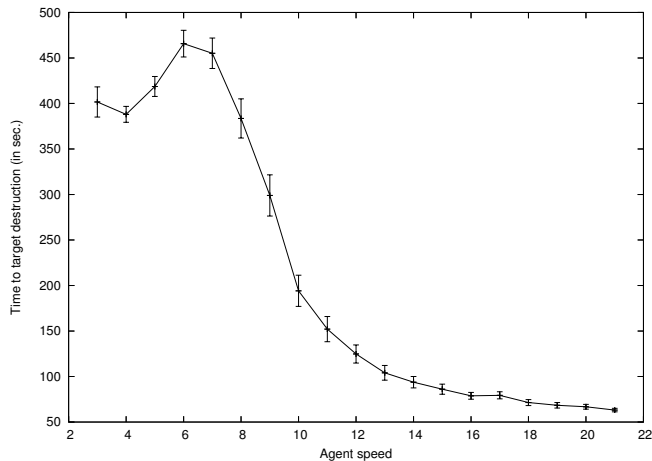


Fig. 3. Simulation results of the area coverage and protection tasks for varying agent speeds (and accordingly varying repulsion ranges ρ) without replenishing destroyed agents.

TABLE I

COMPARISON OF PROPERTIES OF DIFFERENT CONTROL STRATEGIES.

Properties	Navigation Control Schemes			
	Central	Distrib.	Local	Proposed
Complexity	high	med.	low	low
Indiv. perf. guar.	high	high	low	low
Scalability	low	med/high	high	high
Adaptability	low	med/high	med	high
System fault-tol.	low	med.	med/high	high
Model-indep. perf.	low	med.	med/high	high
Computing requir.	high	med/high	med/low	ultra-low
Dig. comm. bandwid.	high	high	med/low	none

in any run; (2) no hostile agent was able to destroy the target in any run; (3) all 9995 hostile agents were destroyed; and (4) the number of alive agents at the end of each 1 million cycle run was higher than the original number because sometimes two hostile agents collided at the same time with the same agent (the average number of excess alive agents at the end of the simulations were 26.9 for $6m/s$, 63.7 for $7m/s$, and 99.11 for $8m/s$).

V. DISCUSSION

Much research in distributed control and robotics has focused on controlling a large groups of autonomous agents. The different control schemes can be roughly put into three categories: *centralized control*, *decentralized control*, and *local control*.

Centralized control is well-known to have a number of problems, such as low fault-tolerance of the system (due to its dependency on many communication links), poor scaling (due to a common sink and command node), and high complexity (due to powerful processing centers), some of which are solved by *distributed control* (e.g., [22], [23], [24], [25], [26], [27]). Both, centralized and distributed navigation control schemes, require digital communication (possibly causing delays, congestion, packet drop, etc.) and often resort to GPS technology for trajectory control. While both paradigms are able to provide performance guarantees for individual agents (by controlling them separately and

providing allocations of specific resources to specific targets), this kind of microscopic control of the entire system is often not needed (e.g., it may not matter which agent intercepts a hostile agent).

Local control takes advantage of this fact, leading to emergent behavior that share many features of general distributed control, but do not suffer from scaling problems (only local neighbors are needed for communication) (e.g., [1], [2], [3], [4], [5], [7], [8], [9], [10], [11], [12], [13], [14]). While most local approaches have low complexity and good fault-tolerance, they still suffer from the consequences of using digital communications (e.g., [21]).

Our proposed approach (see also [1]) is at the ultra-low complexity end of the entire spectrum of local control (see the comparison in Tab. I), leading to very low cost implementations that were previously not feasible: neither digital communication nor GPS are required for navigation, and decisions are based on simple reactive principles that can be implemented in analog hardware and with practically no delay. Of all autonomous swarm systems, the proposed principle, to our knowledge, offers the *lowest cost* and *complexity*, yet at the same time it can successfully accomplish a great variety of tasks and missions (e.g., compare our results to [29], [8], [30], [31]). The advantages of the proposed principle are manifold:

- 1) The principle is **extremely simple** and involves only **local beacon-based interactions** (“emergent behavior”)
- 2) The principle **scales** since there is no digital communication involved for navigation
- 3) The underlying beacon-based mechanisms make the entire system **extremely robust** to individual agent failure, and the performance of the whole system **degrades gracefully** with a decreasing number of agents.
- 4) The principle provides **simple and coarse performance prediction** of the entire swarm in a stochastic sense (rather than each individual agent) even in cases of heterogeneous networks with different types of

agents.

- 5) The principle provides **many special systems for intelligence, surveillance, reconnaissance, target tracking and interception** by simply changing the beacon detection radius and/or making certain agents stationary: mobile sensor networks, multi-agent target detection and tracking, patrolling for fixed surveillance tasks, interception tasks for infra-structure protection, targeting, plume tracking, ad-hoc cellular communication service facilitation, etc.

VI. CONCLUSION

We proposed a novel low-cost beacon-based principle for navigation control for autonomous multi-agent systems that does not require digital communication, thus allows systems to scale, is highly robust, and can achieve high task performance in variety of tasks. In particular, we investigated the utility of the principle for agent swarms in an area coverage and protection task, in which agents need to intercept intruders before they can destroy a target structure. We demonstrated with extensive simulation experiments that there are configurations of the system in which simple swarm agents *without the ability to detect intruders* can successfully protect the target and intercept all intruders. Future work will focus on establishing theoretical properties of the employed principle and simulations will be extended to other application domains, where autonomous agent swarms using the principle might be useful (e.g., providing adhoc communication infrastructure in disaster areas).

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